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Résumé de l'article

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Our applications are conducted on Canadian data. The results reveal some important identification difficulties associated with some of our models, reinforcing the need to use identification-robust methods to estimate such policy functions. Despite these challenges, we are able to obtain fairly comparable point estimates for the real equilibrium interest rate across our different models, and in the case of the best fitting model, also remarkable estimate precision.

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IDENTIFICATION-ROBUST ESTIMATES OF THE CANADIAN NATURAL RATE OF INTEREST*

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Abstract–The natural rate of interest is an unobservable entity and its measurement presents some important empirical challenges. In this paper, we use identification-robust methods and central bank real-time staff projections to obtain estimates for the equilibrium real rate from contemporaneous and forward-looking Taylor-type interest rate rules. The methods notably account for the potential presence of endogeneity, under-identification, and errors-in-variables concerns.

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Introduction

When inflation is at its desired value and output is at potential level, the corresponding real interest rate can be referred to as the natural (or equilibrium or neutral) real interest rate. This variable is a key ingredient in Taylor-type rules and thus plays an important role in the analysis of monetary policy predicated on such rules.

Yet the natural rate is an unobservable entity, and it must somehow be extracted from the data. While various strategies have been proposed for this purpose, including financial-based, dynamic stochastic general equilibrium (DSGE), and time-series-based approaches, the related literature has emphasized the difficulty of getting

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reasonable estimate precision for the obtained measures of the natural rate. As discussed in the next section, some of these difficulties appear to be related to identification problems. The literature also brings to light a second concern with the estimation of the natural rate, which is the sensitivity of the estimated values to the particular specification of the underlying macroeconomic or financial model[1].

In this paper we estimate the natural real interest rate for Canada addressing the two aforementioned issues. First, relying on a Taylor-rule-based approach, we consider a number of different specifications for this macroeconomic model, including, given the importance of forward-looking behaviour for macroequilibrium outcomes, specifications with contemporaneous or forward-looking explanatory variables. Second, we apply identification-robust methodology to these various alternatives. Unlike traditional methods, identification-robust approaches allow for valid inference on parameter estimates even in the presence of identification difficulties.

Of course, with contemporaneous and forward-looking explanatory terms in our equations, endogeneity becomes an important concern. Cochrane (2011) contends that, from a theoretical perspective, Taylor rules within new Keynesian setups are actually predicated on the endogeneity of the right-hand-side variables. That is, in the new Keynesian paradigm, policy-makers set the interest rate to a value that makes it compatible with expected deviations of inflation and output from their respective targets, as well as with any persistence in the equilibrium real rate or in the policy shock, while simultaneously keeping inflation on a determinacy path. This generates a number of restrictions on the model and on the choice of the desired equilibrium values (specially if the latter exhibit persistence) which are somehow implicitly assumed in model applications but never fully articulated. Among the important implications are that (i) the regressors of the Taylor rule are necessarily endogenous, and (ii) lags of regressors in the new Keynesian setup are not valid instruments for conducting any estimations with, specially in models with serial correlation in the shock terms or in the equilibrium real rate.

Yet test statistics based on traditional inference methods are incapable of providing any indication as to whether the instruments used are valid or not. Furthermore, new econometric methods that cater to weak-instruments problems have shown that both limited-information methods (for example, instrumental-variable or generalized method of moments (GMM)) and full information-based methods (such as maximum-likelihood and median-unbiased methods) can lead to spurious rejections and wrong conclusions in the presence of weak instruments (or more generally, under weak identification). In contrast, recently developed identification-robust methods are not only valid whatever the identification status of the model under examination, but they also reveal the extent of statistical identification. For instance, the weaker the instruments, the wider the obtained projections around

1. See, for example, Weber, Lemke and Worms(2007), and Giammarioli and Valla (2004) for surveys on the topic.

the estimated parameters. Similarly, if instruments are completely non-valid, then the methods will reject the model. Indeed, applications of identification-robust methods in macroeconomics have shown that estimate uncertainty is often actually much larger than had been estimated based on traditional validation methods. A recent example is provided by Mavroeidis (2010) who revisits the Clarida, Gali and Gertler (2000) study with identification-robust methods and with the original dataset, and who obtains considerably larger estimate uncertainty than ones reported in the original study[2].

The above discussion emphasizes the need to use identification-robust methods to estimate monetary policy rules with potentially endogenous regressors, a strategy that we follow in this paper. In addition, in our applications, we make use of real-time central bank staff forecasts for the expected terms in the policy equations. Studies such as Orphanides (2001) and Boivin (2006) have stressed the importance of using monetary authority staff real-time forecasts for obtaining accurate representations of historical decisions made by policy makers, and to avoid getting biased estimates from ex post revised data.

The results reveal important identification difficulties associated with some of our models, reinforcing the need to use identification-robust methods to estimate such policy functions. Despite these challenges, we are able to obtain fairly comparable point estimates for the real equilibrium interest rate across the different specifications that we consider. In addition, we demonstrate the merit of using forward-looking information in Taylor rules, showing that such a specification is also able to deliver remarkable coefficient estimate precision.

In the next section we present the theoretical model examined. Section 2 explains the methodology that we apply. Section 3 presents the data and empirical results, and the last dection offers some conclusions.

1. The Model

We make use of the structural form of the model proposed by Clarida, Gali et Gertler (2000) that is both forward-looking and accounts for the tendency of the monetary authority to smooth interest rates across periods. Designating ρ as the degree of smoothing, at any given period the nominal interest rate i_t is thus a weighted average of its lag and of its target rate i_t^* so that $i_t = \rho i_{t-1} + (1-\rho) i_t^*$.

For the target interest rate, it is assumed that policy responds to deviations of expected future inflation from the announced inflation target and of expected future output gap from zero. The corresponding equation is given by:

$$i_t^* = i^* + \beta(E_t \pi_{t,s} - \pi^*) + \gamma(E_t x_{t,q}),$$

2. Other recent examples include Dufour, Khalaf and Kichian (2006), Nason and Smith (2008), Kleibergen and Mavroeidis (2008), Dufour, Khalaf and Kichian (2009), Canova and Sala (2009) as well as Dufour, Khalaf and Kichian (2013).

where π^* is the inflation target, $\pi_{t,s}$ is inflation s periods ahead, $x_{t,q}$ is the output gap q periods ahead, and i^* is the desired nominal rate when the other right-hand-side terms are equal to zero. We further assume that expectations are formed based on information available at time t, and that expected s-period-ahead inflation and q-period-ahead output gap are given by the central bank staff forecasts $\pi^f_{t,s}$, and $\gamma x^f_{t,q}$, respectively, plus unexpected and uncorrelated forecast errors.

Integrating the above specification in the weighted average equation, using the staff forecasts and defining the long run equilibrium real rate by $r^* = i^* - \pi^*$, we obtain the following expression for the nominal interest rate:

$$i_t = (1-\rho)[r^* - (\beta-1)\pi^* + \beta\pi^f_{t,s} + \gamma x^f_{t,q}] + \rho i_{t-1} + \varepsilon_t. \quad (1)$$

The real rate is thus assumed to be a constant (with the assumption that it is not determined by monetary factors) and the residual ε_t, which is a sum of orthogonal forecast errors, is assumed to be independently and identically distributed. As discussed in Cochrane (2011), some of these assumptions may be restrictive and need to be checked. Therefore, in later sections, we consider regressions over subsamples to see if the neutral rate estimates change over time. In addition, in all our regressions we use test statistics that account for possible serial correlation in the residual term.

Given the announced inflation target of 2 per cent in Canada, and imposing the structural constraints implied by equation (1), it is possible to obtain estimates for the parameters ρ, β and γ, as well as for the long run real rate r^*. For a set of instruments that are available and known to the policy-maker at the time the interest rate is set, and presumed to be orthogonal to the time t policy shock, as in Clarida, Gali and Gertler(2000), one can use GMM to estimate the unknown parameters of the model. However, as explained previously, estimates and over-identification tests based on GMM are useful only if the instruments are valid, something that is impossible to assess from the GMM outcomes themselves. Accordingly, we use identification-robust methods to conduct our estimations and tests. The next section discusses these methods and related issues.

2. Identification-Robust Methods

When taken to the data, econometric models with expectations-based regressors (often derived from optimization-based foundations) are generally confronted with endogeneity, errors-in-variables problems, and various parameter nonlinearities related to underlying structural constraints. While, given an instrument set, it is possible to obtain orthogonality conditions from these models that can be used for instrumental variable (IV) or GMM estimation methods, the presence of weak instruments can lead to weak identification with these models[3].

3. The intuition for the use of identification-robust methods and the explanations regarding the methodology are largely based on Dufour, Khalaf and Kichian (2006, 2009, 2009b). For more information and for recent macroeconomic applications see Mavroeidis (2004, 2005), Dufour, Khalaf and Kichian (2006), Nason and Smith (2008), Kleibergen and Mavroeidis (2008), Canova and Sala (2009), and Dufour, Khalaf and Kichian (2009, 2009b).

Weak identification is a serious problem because it causes the breakdown of standard asymptotic procedures such as IV-based *t*-tests, overidentification *J*-tests and Wald-type confidence intervals. In other words, such standard tests can be unreliable and, indeed, spurious model rejections can occur even with large data sets.

In principle, non-identification should lead to diffuse confidence sets that can alert the researcher to the problem. Unfortunately, if traditional Wald-type methods are applied when estimating weakly identified parameters, the opposite often happens, with tight confidence intervals obtained around wrong values. Therefore, not only is the estimation uncertainty understated (sometimes severely), but also intervals will fail to cover the true parameter value. Furthermore, in view of the tightness of the intervals, both problems will go unnoticed.

Identification-robust generalized Anderson-Rubin (GAR) methods are immune to these issues because they are valid regardless of the identification status of the model being examined[4]. GAR methods consist in specifying an auxiliary regression set-up that translates the model facing identification problems into the regular regression framework (that does not face these concerns) while maintaining its structural foundations. Testing in the context of the auxiliary regression then amounts to testing the original model. Furthemore, parameters are tested jointly and confidence sets can be obtained. From here, extremely-wide confidence sets reveal identification difficulties. In addition, if all economically-sound values of the model's deep parameters are rejected at some chosen significance level, the confidence set will be empty and the model is thus rejected. The latter is an identification-robust alternative to the standard GMM-based *J*-test

Our methodology can be described more formally as follows. Consider a nonlinear equation of the form:

$$\mathcal{F}_t(\mathcal{Y}_t,\vartheta) = \mathcal{U}_t, \quad t = 1,\ldots,T, \tag{2}$$

where $\mathcal{F}_t$, $t = 1,\ldots,T$ are scalar functions that may have a different form for each observation, ϑ is an m x 1 vector of unknown parameters of interest, $\mathcal{Y}_t$ is the n x 1 vector of observed variables and $\mathcal{U}_t$ is a disturbance with mean zero. The vector $\mathcal{Y}_t$ includes the exogenous and endogenous variables of the model. The objective is to test the hypothesis:

$$H_0 : \vartheta = \vartheta_0.$$

If H_0 holds true, then $\mathcal{F}_t(\mathcal{Y}_t,\vartheta_0) = \mathcal{U}_t$. Thus, if Z_t is a k x 1 vector of exogenous or predetermined variables such that $k \geq m$, then the coefficients of the regression

$$\mathcal{F}_t(\mathcal{Y}_t,\vartheta_0) = Z_t'\varpi + \varepsilon_t \tag{3}$$

4. In other words, these are methods where error probabilities (e.g. test size, confidence level) can be controlled in the presence of endogeneity, nonlinear parameter constraints and identification difficulties; see the literature cited in Dufour, Khalaf and Kichian (2006, 2009) for the theoretical explanations.

should be close to zero. Hence, H_0 in the context of (2) can be tested by assessing

$$H'_0 : \varpi = 0 \tag{4}$$

in the context of (3). Z_t can be viewed as a vector of instruments, which may include the exogenous variables in $\mathcal{Y}_t$; Equation (3) may be viewed as an auxiliary or artifical regression and the test of H'_0 in the context of (3) an auxiliary or artificial regression test for H_0. Rewriting the latter in matrix form where

$$\mathcal{F}(\mathcal{Y},\vartheta_0) = [\mathcal{F}_1(\mathcal{Y}_t,\vartheta_0),\ldots, \mathcal{F}_2(\mathcal{Y}_t,\vartheta_0)]', \tag{5}$$

$$\mathcal{Y} = [\mathcal{Y}_1,\mathcal{Y}_2,\ldots, \mathcal{Y}_T]', \tag{6}$$

$$Z = [Z_1,Z_2,\ldots, Z_T]', \tag{7}$$

the F-statistic for H'_0 is given by

$$\mathcal{T}(\vartheta_0) = \frac{\mathcal{F}(\mathcal{Y},\vartheta_0)'\left(I - M[Z]\right)\mathcal{F}(\mathcal{Y},\vartheta_0)/k}{\mathcal{F}(\mathcal{Y},\vartheta_0)'M[Z]\mathcal{F}(\mathcal{Y},\vartheta_0)/(T-k)} \tag{8}$$

$$M[Z] = I - Z(Z'Z)^{-1}Z'. \tag{9}$$

If Z and $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_T$ are independent, the matrix Z has full column rank and $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_T$ are *i.i.d.* homoscedastic normal, under the null hypothesis (4), $\mathcal{T}(\vartheta_0)$ follows a central Fisher distribution with degrees of freedom k and $T-k$. The latter exact result may be relaxed leading to the standard χ^2 based distribution compatible with classical least-squares.

Allowing for departures from the *i.i.d.* error hypothesis, the test we invert is based on a Wald-type statistic with Newey-West autocorrelation consistent covariance estimator given by:

$$AR-\mathrm{HAC}\left(\vartheta_0\right) = \mathcal{F}(\mathcal{Y},\vartheta_0)'Z(Z'Z)^{-1}\hat{Q}^{-1}(Z'Z)^{-1}Z'\mathcal{F}(\mathcal{Y},\vartheta_0) \tag{10}$$

$$\hat{Q} = \frac{1}{T}\sum_{t=1}^{T}\hat{u}_t^2 Z_t Z_t' + \frac{1}{T}\sum_{l=1}^{L}\sum_{t=l+1}^{T} w_l \hat{u}_t \hat{u}_{t-l}\left(Z_t Z_{t-l}' + Z_{t-l}Z_t'\right)$$

$$w_l = 1 - \frac{l}{L+1}$$

where $\hat{u}_t$ is the OLS residual associated with the artificial regression (3), and L is the number of allowed lags. Although, for simplicity, our notation may not clearly reflect this fact, it is worth emphasizing that $\hat{Q}$ is a function of ϑ_0 so the minimum distance-based test we consider involves continuous updating of the weighting matrix.

The above procedure describes the inversion of an identification-robust test. Inverting a test produces the set of parameter values that are not rejected by this test. In addition, the least rejected parameters are the so-called Hodges-Lehmann point estimates (see Hodges and Lehmann, 1963; 1983; and Dufour, Khalaf and Kichian, 2006).

In practice, test inversion is performed numerically. A $(1-\alpha)$ level confidence set is constructed by collecting the ϑ_0 that, given calibrated parameters, are not rejected by the above tests at level α. For this purpose we conduct a grid search over the economically meaningful set of values for the structural parameters, sweeping the choices for the parameters comprising the vector ϑ_0. For each parameter combination choice, the identification-robust test statistic is applied, and the associated p-value is calculated from the $\chi^2(k)$ null distribution. Collecting those vector choices for which the p-values are greater than a test level α constitute a joint confidence region with level $1-\alpha$. Individual confidence intervals for each parameter can then be obtained by projecting the latter region (i.e. by computing, in turn, the smallest and largest values for each parameter included in this region). A point estimate can also be obtained from the joint confidence set. This corresponds to the model that is most compatible with the data, or, alternatively, that is least rejected, and is given by the vector of parameter values with the largest p-value.

3. Empirical Results

3.1 *The Data*

We conduct our estimations on quarterly Canadian data over the sample 1989Q1 to 2005Q4. The start of our sample is chosen to be comformable with the beginning of the inflation-targeting era in Canada[5]. Real-time staff forecasts for inflation and the output gap, both current and forward-looking, are obtained from the Bank of Canada staff projection model's forecasts. Table 1 of the Appendix presents some basic statistics on selected items. All the variables used are measured in annual percentages.

In Canada, the announced inflation target is designated in terms of total consumer price index (CPI) inflation. However, the operational inflation rate of the monetary authority is the core inflation rate[6]. Accordingly, our estimations are carried out using the core inflation rate for π_t which is defined as the log difference (annualized) of the core index p_t. We use the 30-day Canadian T-bill rate as the Canadian nominal interest rate and the Bank of Canada staff projection model's output gap measure.

We also require a number of instruments. We use lags of four variables including the change in Canadian employment, the US growth rate, as well as two financial

5. Given the policy-sensitive nature of the neutral interest rate, and given the limited availability of the real-time central bank staff forecasts, the sample ends in 2005Q4.

6. The core price index measure is obtained by excluding the eight most volatile components as well as the effect of indirect taxes from the Canadian CPI.

variables. The latter are the term spread, defined as the difference between the 5-to-10 year bond yields and the 30-day treasury bill rate, and the change in a composite leading indicator for Canada obtained from Statistics Canada.

3.2 *Model Estimations and Results*

3.2.1 *Constant Equilibrium Real Rate Estimates*

Equation (1) is estimated using the generalized Anderson-Rubin methods described above for the different combinations for $s,q = (0,1)$. This includes the $s = 1$, $q = 0$ baseline case considered in Clarida, Gali and Gertler(2000); that is, a time $t + 1$ expected inflation term and a contemporaneous value for the expected gap. In all our estimations, the inflation and gap terms are considered to be endogenous.

We make use of two instrument sets Z_1 and Z_2 to conduct our estimations. Following Cochrane (2011), we avoid the use of lagged endogenous variables and instead include lags of each of: the term spread, the change in employment, the US growth rate, as well as the change in the Canadian composite leading indicator[7]. Set Z_1 includes one lag of these variables while set Z_2 includes two lags. Estimations are conducted imposing the structural constraints of the model. In all cases, the test that is applied is the AR-HAC test and significance refers to a five per cent test level. Four lags are used in the Newey-West heteroscedasticity and autocorrelation-consistent covariance estimator. For our numerical grid search, the search space for r^* is (0.00, 6.00), for β it is (0.00, 6.00), for γ (-1.00, 2.00), and for the lag coefficient p, we consider the space (0.50, 0.98)[8]. The grid increments are 0.13, 0.20, 0.10, and 0.02, respectively. Finally, we set the inflation target π^* to 2 per cent.

Estimation and test results are reported in Table 2 of the Appendix. In the case of each model, we report the point estimates of the structural parameters, and in parentheses we report its smallest and highest values in the confidence set. We refer to these as the estimate projections. We also report the root mean squared error (RMSE) of the residuals obtained by fixing model coefficients at their point estimates and substracting the fitted values from actual values for the dependent variable.

From the table we see that obtained estimates are fairly similar across models, with the exception of the ($s = 1$, $q = 1$) with Z_2 model that yields different point estimate values for three of the parameters, and the ($s = 1$, $q = 0$) with Z_2

7. Term spreads were shown to have useful forecasting capacity for macroeconomic variables (see, for instance, Stock and Watson, 2002, 2003). Lags of the change in the leading indicator (see Statistics Canada for details) are included for similar reasons. The importance of the US for the Canadian economy dictates the inclusion of lags from the US growth rate among the instruments. Finally, lags of the change in employment are included to capture, among other things, any frictions that might be present in the labour market.

8. Note that we remain agnostic regarding determinacy and do not impose that the coefficient on inflation be greater than one.

specification which is rejected by the data at the 5 % level. Furthermore, the conclusions that we can draw from our results are overall in line with those reported on the Canadian policy rule in the literature[9]. In particular, the coefficient on inflation in our considered sample is greater than the coefficient on the output gap by a substantial margin, and there is evidence for smoothing in interest rates.

Based on the retained specifications we find that point estimates for the equilibrium real interest rate fall within 30 percentage points, that is between the values of 2.4 and 2.7 percent. These values are much lower than those obtained by Lam and Tkacz (2004) where a calibrated DSGE set-up for Canada was used. But while this difference in results could be due to many things, including the use of real-time versus revised data, and to the important differences in model specifications, since our methodology also provides projections around point estimates, we will be able to evaluate which other values for the natural rate are also compatible with the data at the 5 % level.

The projections are reported in parentheses and provide information on the precision of the obtained estimates. Two main findings can immediately be observed from these. The first is that the most precisely estimated model is the forward-looking ($s = 1$, $q = 1$) model that uses instrument set Z_2. While the remaining models are able to yield fair precision around a few of their coefficient estimates, the purely forward-looking equation in conjunction with the Z_2 instrument set is remarkably well estimated, yielding fairly tight projections around all of its parameters.

Second, estimation results, notably those based on the models with one or more contemporaneous regressors, reveal evidence of identification difficulty. In particular, the projections for the p estimates sometimes hit the lower bound value of 0.50 of the search space for this parameter[10]. As for the other parameters, projections are somewhat wide, except for the output gap which is fairly precisely estimated but which we also find to be insignificant. Adding information to the model set-up in the form of more lags in the instrument set sometimes helps to improve precision and other times not. Comparing the ($s = 0$, $q = 0$) model outcomes based on Z_1 and Z_2, we see that projections actually widen for three of the four parameters when more information is incorporated into the model (the projection around the natural rate does not widen but shifts a little). On the other hand, comparing the ($s = 1$, $q = 1$) models across the instrument sets shows a substantial tightening of projections with Z_2. These examples further illustrate the importance of using

9. See, for example, Dib (2003).

10. One reason for why point estimates sometimes hit the lower bound for this parameter could be that a value lower than 0.50 should have been considered within the search space, compatible with less policy smoothing by the monetary authorities. Yet while the latter is theoretically possible, it is not very likely considering the traditionally conservative attitude of the central bank over the years and given evidence presented in other studies supporting the view of high smoothing. Another explanation could be that the adopted specification in these particular cases is not entirely suitable, forcing some parameters to take on values that are outside economically reasonable ranges. In particular, we see that ρ hits the lower bound value only in the cases where the other regressors are both assumed to be contemporaneous.

identification-robust methods where, depending on the model specification, the same instruments may either weaken statistical identification or strenghten it. Therefore, rather than relying on Wald-test-based confidence intervals that are only valid when identification is strong, our methods allow us to guage the true estimate uncertainty when in fact identification is weak, and thus prevents us from potentially drawing wrong conclusions.

The table also shows that the root mean square error associated with the model in-sample fit is much smaller in the presence of forward-looking endogenous inflation or output gap terms. For instance, for the models estimated with instrument set Z_1, the RMSE is almost halved, dropping from a value of 2.255 to 1.389. Similarly, for the model estimations relying on Z_2, the RMSE decreases from 2.263 to 0.827. Therefore, it would appear that it is important to use forward-looking based rules (at least for Canadian data), as the theory suggests. Thus, even when more information is considered in the model specifications (via the instrument set Z_2) it is the combination of this with the forward-looking element that yields the improvements discussed above.

Based on the best estimated model, some important features can be learnt about the economic and policy environment in Canada. The point estimate for the coefficient on the expected inflation gap term is 3.40, and the associated projection region is (3.0 to 3.6), putting this number firmly in the above one region and providing evidence of the strong inflation-targeting focus of the monetary authority in Canada over the examined period. At the same time, we see that the coefficient on the output gap is relatively small (0.20), though it is quite precisely estimated and is statistically significant (zero being excluded from its projection region)[11]. In addition, we find stong evidence for interest rate smoothing by the monetary authority, with a value of 0.84 for the point estimate and a fairly tightly estimated uncertainty associated with this parameter, namely the range 0.77 to 0.84[12]. As for the projections around the natural rate estimate, these are also quite tight, ranging from 2.50 and 3.10, and ruling out the high values of around 5 reported by Lam and Tkacz (2004). This precision is substantially better than ones obtained on the basis of the models with contemporaneous regressors, where with instrument set Z_1 the range is (1.8, 3.6), and with Z_2 it is (2.0, 3.9).

11. Note that Mavroeidis (2010) finds substantially wider projections for his reduced-form Taylor rule parameter estimates for US data and for the baseline Clarida, Gali and Gertler(2000) specification with lagged endogenous variables as instruments. In particular, his projections on the inflation variable include regions with values both less than and above one.

12. The reader is reminded of the fact that, unlike usual Wald-based confidence intervals, identification-robust projections are not symmetric. This means that two adjacent values in the projection region may have very different p-values (above 0.05) associated with them. This is why, point estimates (which have the highest p-value in the projection) may end up at the very edge of the projection region and not necessarily in its centre.

3.2.2 *Assessing Parameter Stability*

One issue that has been discussed in the literature is whether the natural rate has varied over time. For example, Laubach-Williams(2003) suggest that, for the US, this rate declined from 4.5 percent in the mid-sixties to around 2.5 percent in the mid-seventies. However Clark-Kozicki(2005) document some important difficulties in obtaining precision of estimates for a time-varying natural rate, particularly when the variation in the latter is related to changes in the trend growth rate. Among these are the problems associated with one-sided filtering and sensitivity to initial values within the Kalman-filtering-based state space representations, as well as sensitivity to the assumed volatility of the trend growth rate. Revisions in the data over time also contribute to the difficulty of precisely pinning down parameter estimates for the natural rate.

We follow a different approach to assess time-variation of the real rate. We use rolling fixed window regressions to study the possible evolution in the value of this parameter over time. Taking our first subsample as ending in the fourth quarter of 1999, and focusing on the ($s = 1, q = 0$) and ($s = 1, q = 1$) specifications, we estimate the model coefficients over successive subsamples where the first and last points are moved forward in time by one observation each. In all cases we continue to apply identification-robust methods while preserving the structural constraints of the model, and we calculate the in-sample RMSEs for each case. The search spaces are the same as they were for the full sample case but for computational convenience we use slightly coarser grid increments (that is, 0.30 for r^* and β, 0.15 for γ, and 0.024 for p).

Table 3 documents the obtained estimates of the equilibrium real rate obtained with the instrument set Z_1, along with the associated projections and root mean squared errors[13]. From here we can draw a number of conclusions. The first is that, with a few exceptions, results (point estimates, projections and even RMSE's) are fairly comparable across the two model specifications. Looking more closely, we find that one model occasionally yields more precision than the other over a given subsample, and sometimes it also has a lower in-sample RMSE. For instance, for the period ending in 2005: 01, the projection around the r^* estimate is tighter with the purely forward-looking model, and the associated RMSE is lower. However, differences in obtained point estimates across a given subsample are not significant as projections of the one model include the point estimate of the other model.

Second, we find estimate uncertainty to be quite important across time and that projections often hit the upper bound of the parameter search space. Thus, the methods that we apply are able to statistically rule out very low values for the natural rate, but often not values that are closer to 6. Out of the 25 subsamples considered, only 9 of the projections are bounded both from above and below in the case of the purely forward-looking model, and only 10 in the case of the other

13. We found that the models were rejected over many of the subsamples when we used the set Z_2.

specification. Nevertheless, we also observe certain subsamples where r^* is much more precisely estimated. For example, for the sample ending in the third quarter of 2005, the obtained projections are (2.4, 3.6) and (2.7, 3.6) for the two models.

Finally, we observe that point estimates for the natural rate vary between 3.0 and 4.5 per cent over our subsamples. For example, it appears that there has been a decline from 3.6 to in the first few subsamples to values of 3.0-3.3 over the next few subsamples, subsequently followed by an increase to values of 3.9-4.5 for the samples ending in early 2004. However, it should be emphasized that these observations are not statistically verified since (i) proper testing would require projections from a joint multiple-sample identification-robust confidence set, which is outside the scope of the present study, and (ii) r^* point values of early subsamples that are relatively well estimated fall within the projections of later subsamples where the natural rate is also well estimated.

Conclusion

In this paper we revisited the empirical measurement of the equilibrium real rate in Canada by using real-time data, notably central bank staff forecasts, and by using both contemporaneous and forward-looking Taylor type rules. In addition, identification-robust estimation and test methods were used, which, in addition to preserving the structural constraints of the considered models, also account for endogeneity, underidentification, and errors in variables concerns.

We found the results to be generally similar across models, but the best in-sample fit and the most precise estimates were obtained for a purely forward-looking model. The importance of using identification-robust methods was also demonstrated, as some of the projections for the estimates turned out to be quite wide, hitting either the upper or lower boundary of the parameter search space, and as adding information to the instrument set improved statistical identification in some cases but worsened it in others.

APPENDIX

TABLE 1

BASIC STATISTICS

VARIABLE	MEAN	STANDARD ERROR
Short term rate	5.37	3.12
Spread	1.07	1.58
Growth rate	2.62	2.44
CPI Inflation	2.35	2.31
Core Inflation	2.15	1.93

TABLE 2

MODEL ESTIMATION AND TEST RESULTS

Horizon	r^*	β	γ	p	RMSE
Instrument set Z_1					
$s = 0, q = 0$	2.60 (1.80, 3.60)	2.80 (2.20, 4.00)	0.00 (0.00, 0.20)	0.50 (0.50, 0.79)	2.255
$s = 1, q = 0$	2.60 (1.60, 3.40)	2.80 (2.20, 4.00)	0.00 (-0.10, 0.10)	0.55 (0.50, 0.77)	1.389
$s = 1, q = 1$	2.60 (1.60, 3.60)	2.80 (2.20, 4.00)	0.00 (-0.10, 0.20)	0.55 (0.50, 0.80)	1.389
Instrument set Z_2					
$s = 0, q = 0$	2.40 (2.00, 3.87)	2.80 (2.00, 5.40)	0.00 (-0.10, 0.40)	0.50 (0.50, 0.87)	2.263
$s = 1, q = 0$	Model is rejected				
$s = 1, q = 1$	2.67	3.40	0.20	0.84	0.827
	(2.53,3.07)	(3.00, 3.60)	(0.10, 0.20)	(0.77, 0.84)	

The instrument sets Z_1 and Z_2 include one, and two lags, respectively, of each of: the term spread, the change in employment, the US growth rate, as well as the change in the Canadian composite leading indicator. For the first three (s,q) cases, first and second lags are considered; for the last two, second and third lags are used, to account for the MA(1) structure of the errors.

TABLE 3

EQUILIBRIUM RATES AND PROJECTIONS; ROLLING FIXED WINDOW ESTIMATES

	($s = 1$, $q = 1$)			($s = 1$, $q = 0$)		
Period End	$\hat{r}^*$	**Projections**	**RMSE**	$\hat{r}^*$	**Projections**	**RMSE**
1999: 04	3.6	(2.7, 6.0)	0.978	3.6	(3.0, 6.0)	0.977
2000: 01	3.6	(2.7, 6.0)	1.003	3.6	(2.7, 6.0)	1.002
2000: 02	3.6	(2.7, 6.0)	1.112	3.6	(2.7, 6.0)	1.111
2000: 03	3.6	(2.7, 6.0)	1.224	3.6	(2.7, 6.0)	1.167
2000: 04	3.6	(2.7, 6.0)	1.057	3.6	(2.7, 6.0)	1.056
2001: 01	3.3	(2.7, 6.0)	0.781	3.3	(2.7, 6.0)	0.781
2001: 02	3.0	(2.7, 6.0)	0.747	3.3	(2.7, 5.7)	0.772
2001: 03	3.0	(2.4, 5.7)	0.769	3.0	(2.4, 4.5)	0.769
2001: 04	3.3	(2.7, 5.1)	0.881	3.3	(2.7, 5.4)	0.882
2002: 01	4.2	(2.7, 6.0)	1.880	4.5	(2.7, 6.0)	2.077
2002: 02	3.9	(2.4, 6.0)	0.848	4.2	(2.4, 6.0)	0.950
2002: 03	4.5	(2.1, 6.0)	0.966	4.5	(2.4, 6.0)	0.967
2002: 04	3.6	(2.7, 6.0)	0.993	3.3	(2.4, 5.7)	0.977
2003: 01	3.6	(2.7, 6.0)	0.909	3.6	(2.4, 5.4)	0.907
2003: 02	3.6	(2.1, 6.0)	0.909	3.6	(2.7, 6.0)	0.991
2003: 03	3.9	(2.7, 5.4)	1.680	3.9	(2.7, 5.4)	1.881
2003: 04	4.5	(2.4, 5.7)	0.966	4.5	(2.4, 5.7)	0.967
2004: 01	3.9	(2.7, 6.0)	1.680	3.9	(2.4, 6.0)	1.881
2004: 02	3.9	(2.7, 6.0)	1.680	3.9	(2.4, 6.0)	1.780
2004: 03	3.6	(3.0, 4.8)	1.172	3.6	(2.7, 4.8)	1.170
2004: 04	3.6	(2.7, 4.8)	0.993	3.3	(2.7, 4.8)	0.895
2005: 01	4.2	(2.7, 5.1)	0.860	3.3	(0.0, 5.1)	0.895
2005: 02	3.6	(2.4, 5.1)	0.760	3.6	(0.0, 5.1)	0.762
2005: 03	3.3	(2.4, 3.6)	0.756	3.0	(2.7, 3.6)	0.886
2005: 04	4.5	(1.8, 6.0)	0.697	3.0	(2.1, 6.0)	0.757

NOTE: Instrument set Z_1 is used for the above estimations. The first sample starts in 1989: 01. RMSE refers to in-sample root mean square error.

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