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Résumé de l'article

L'insistance de Peirce à montrer l'antériorité et l'indépendance de la logique et des mathématiques sur les sciences naturelles rencontre de sérieuses objections : l'affirmation, en premier lieu, selon laquelle tout savoir est scientifique semble laisser entendre que toute justification des principes de la méthode scientifique présuppose la légitimité de ces principes, s'avère un cercle vicieux; sa prétention, en second lieu, à affirmer que les vérités de la logique et des mathématiques se tiennent en dépit des faits concernant le monde actuel, semble difficile à cadrer avec l'insistance qu'il met à affirmer qu'elles sont établies par observation, suivant ainsi la méthode expérimentale; et, finalement, sa vision que voulant que logique et mathématiques soient des sciences au même titre epistémologique que toutes les autres, semble en contradiction avec sa vision voulant que les résultats, en mathématiques et en logique, sont plus fiables que ceux des sciences naturelles. J'argue que les réponses de Peirce à ces objections reposent sur sa théorie des icônes. Si cela s'avère, cette théorie est au coeur de son épistémologie et résulte en une vue originale des mathématiques et de la logique qui a été largement négligée.

Peirce and the Logic of Diagrams

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Charles Peirce views logic as the branch of philosophy that studies the 'general conditions of the attainment of truth' (NEM 4 : 196).[1] He insists that the principles of logic must be established independently of discoveries in the natural sciences (W1 : 61).[2] As he sees it, findings in the natural sciences are justified by logical principles of inquiry and it is viciously circular to appeal to conclusions warranted by logical principles as reasons for accepting these same principles.[3] On his view, logic rests on mathematics and phenomenology, both of which are epistemologically prior to, and independent of, knowledge in psychology, biology or physics (W1 : 422).[4]

Peirce's insistence that logic and mathematics are prior to the natural sciences faces powerful objections. In the first place, he thinks all knowledge is obtained by means of scientific inquiry – logic and mathematics included. But if this is so, it seems any knowledge used to justify the logical principles of scientific inquiry presupposes the legitimacy of those principles and thus is circular. Secondly, Peirce claims that the truths of logic and mathematics hold regardless of how things are in the actual world. This seems hard to square with his view that '[a]ll knowledge whatever comes from observation' (CP 1.238). How is knowledge that is not contingent on what the actual world is like to be grounded in what is observable? Finally, if logic and mathematics are experimental sciences, it seems their results have the same epistemological status as findings in the natural sciences. How then can Peirce claim that results in logic and mathematics are more secure than those in the natural sciences?

I argue that Peirce has answers to these three objections and that his answers rest on his account of diagrammatic reasoning, an account that is based heavily on his theory of icons. If this is correct, then his theory of icons is central to his epistemology and yields a view of logic and mathematics that is both original and largely overlooked.

1. A Non-Circular Justification of Logic

Peirce holds that since all knowledge is scientific knowledge any justification of the logical principles of scientific inquiry must draw on results obtained through scientific inquiry. However, he denies that a scientific justification of the logical principles of inquiry must assume the truth of those principles in a way that is viciously circular. As he sees it, logical principles of inquiry rest on principles of mathematics and mathematical knowledge is prior to, and independent of, theories in logic. Thus, he thinks, a non-circular scientific foundation for logic is not too much to ask.

Mathematics, for Peirce, is the science of drawing deductive inferences. For him, mathematical results are derived by performing operations on configurations of signs so as to reveal their formal relations (CP 4.530). Whether carrying out arithmetical calculations (using long division, say), solving equations in algebra, or constructing geometrical proofs with a ruler and compass, mathematicians first make inscriptions, then modify them according to rules for introducing and eliminating (or erasing) further signs, and, finally, draw general conclusions from what results.

On this view, cases of reasoning that one might take to be purely logical or conceptual are in fact mathematical. Consider a standard syllogism for example :

All humans are mortal,
<u>Socrates is human</u>,
Thus, Socrates is mortal.

At first blush this does not seem to be a case of mathematical reasoning. However, the logical relations involved in this inference can be represented in the notation of Boolean algebra – which expresses premises as equations and rules of inference as algebraic operations. This same logical structure can likewise be depicted in Euler's system of diagrams – in which rules for constructing and transforming geometrical figures are used to illustrate the logical relations among terms. Peirce thinks this shows that the process of syllogistic reasoning reduces to the performance of certain operations on signs and thus qualifies as mathematical. While other systems of deduction, including Peirce's own system of logical graphs, are more powerful than syllogistic logic, he thinks they still comprise rules for carrrying out various procedures on signs. Hence, for him, deductive reasoning is mathematical (and vice versa).

But if mathematical inquiry involves reasoning, how can Peirce claim it provides a non-circular justification for logic? If mathematical proofs are deductive, as Peirce claims, how can they be prior to, and independent of, logical principles?

In answering this question, Peirce distinguishes between justifying a conclusion deductively in mathematics – that is, uncovering relations among signs by performing operations on diagrams – and justifying a conclusion by appeal to a logical theory of deduction. Although theories of deductive inference are notoriously controversial, Peirce maintains that the deductions involved in mathematics are not. The legitimacy of the reasoning involved in mathematics is evident to all inquirers, even when they hold different theories of what that legitimacy consists in. Similarly, ordinary people grasp the validity of deductive syllogisms and instances of modus ponens, without first having to know the rules that underly them. Peirce thinks the fact that proofs in mathematics are compelling and do not appeal explicitly to principles of the theory of deduction shows that mathematics is independent of, and prior to, logic.

When justifying a theory of inquiry, Peirce claims logicians appeal to mathematical reasoning – logical principles of inquiry are, he thinks, justified deductively. But, he insists, there is no circularity in this so long as the mathematical arguments relied upon in logic do not presuppose acceptance of the logical principles they are used to defend. Mathematical reasoning provides a non-circular foundation for logic so long as it does not rely on premises drawn from the theory of deductive reasoning.

It might seem that on Peirce's view the justification of logical principles by mathematical reasoning remains circular in virtue of the so-called logocentric predicament. The worry is that we must still assume the legitimacy of deductive (*i.e.* mathematical) reasoning in any justification of the theory of deductive reasoning and that this is viciously circular. However, Peirce sees no problem to remedy here. For him, justification is a matter of advancing reasons in support of conclusions and to demand a justification of logical principles that does not involve reasoning is incoherent. The worry about circularity merely draws attention to the banal fact that there are no rational grounds for beliefs outside the scope of reason itself.[5]

2. Mathematics and Logic as Independent of What Is Actual

Peirce maintains that the truths of logic and mathematics hold regardless of what the actual world is like. Yet he also insists that "[a]ll knowledge whatever comes from observation" (CP 1.238), logic and mathematics being no exception (CP 2.227 and 5.411f.). But how can observation yield anything other than knowledge of the actual world?

And how are the truths of mathematics and logic – truths typically thought to be *a priori* and independent of empirical matters of fact – to be justified by observation, if, as Peirce says, they are not contingent on how things are in the actual world?

On Peirce's view, the truths of logic are independent of what is actual because they are justified by mathematics and mathematical results are arrived at through operations performed on signs without regard for whether those signs are true or false. Logic and mathematics remain observational sciences, however, because they rely on the experimental method. Both points merit further explanation.

Regarding the the independence of mathematics and logic from what actually obtains, Peirce holds that reasoning in mathematics (the reasoning on which logic depends) is aimed at determining what would be the case in any world in which a set of assumed premises is true, regardless of whether these premises are true of the actual world. Given the definition of a triangle as a three-sided, enclosed, plane figure, mathematical reasoning shows, for example, that the interior angles of any triangle must add up to 180°. However, this result holds whether or not there are any triangles in the actual world. Likewise, given as assumptions that all humans are mortal and Socrates is human, mathematical reasoning shows that Socrates must also be mortal. But this conclusion follows whether or not there are any humans, whether or not they are mortal and whether or not Socrates is among them. According to Peirce, mathematics is the study of the forms of relations among configurations of signs – or 'diagrams' as he calls them – and these relations obtain whether or not these signs represent the way things actually are.

As for the claim that mathematics and logic are observational sciences, Peirce argues that the method of inquiry in mathematics (and by extension, logic) is no different from that used in physics or chemistry (4.530). A physicist or chemist works with various kinds of apparatus while the mathematician works with diagrams. Still, the procedures are the same in both cases. Consider the case of the syllogism represented using Euler diagrams noted earlier. The validity of this inference is tested by drawing an image representing a hypothetical state of affairs in which the premises are true. More specifically, a circle is drawn to represent the class of all mortals. Another circle, this time representing the class of human beings, is drawn within the one representing the class of mortals. The resulting image depicts a possible world in which all humans are mortal. Finally, a mark denoting Socrates is added so as to represent him as being a member of the class of human beings in this possible world. It is observed from the diagram that is produced by carrying out these operations that Socrates is mortal (see Figure 1).

The validity of this inference is tested by trying to find a configuration of signs that represents Socrates as something other than mortal in a possible world in which he is human and all humans are mortal. Since it is evident from Figure 1 that this cannot be done, the inference is valid. If the premises are true, the conclusion must also be true.

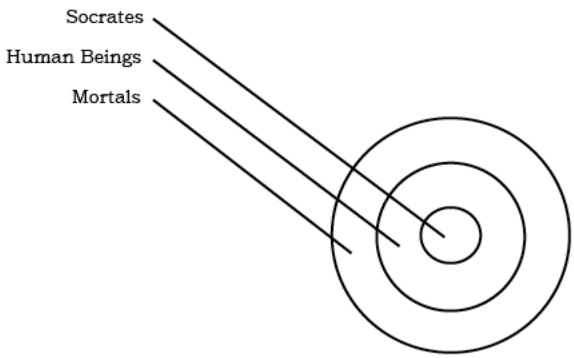

Figure 1

Notice, on this account, what testing the validity of this inference involves. Objects in the actual world (in this case marks on a page) are arranged in accordance with general procedures (here, the rules for constructing Euler diagrams). The particular diagram is itself a tangible object open to public inspection and the procedures performed on it are repeatable with other diagrams of the same type. The effect produced through the performance of the operations on the diagram is observed and it is concluded on the basis of what is observed that the same result would occur in all cases in which the same procedures are carried out.

Peirce thinks that the method of inquiry followed in the case of logic is the same as that used in laboratory science. In one version of his *Experimentum Crucis*, for example, Newton arranges objects in the world so that a beam of white light is directed through a prism to produce a spectrum of colours. He then isolates a single band of light in this spectrum and directs it through a second prism. He observes the result produced by this arrangement of objects – that the colour of the second beam is unaffected by its being refracted – and concludes that the nature of light is such that it would be reproduced in any other case in which the same experiment is engineered.[6] In mathematics inquirers are out to study abstract logical relations, while in the natural sciences they are out to study abstract laws. But in both cases, Peirce insists, inquirers are engaged in performing experiments and discerning their lawful effects.

It is important to note that for Peirce there is more to the claim that mathematics and logic are experimental sciences than the merely verbal manoeuvre of stretching the term 'experiment' to cover the manipulations of symbols performed by logicians and mathematicians. He argues that viewing mathematics as an experimental science is crucial to understanding the discovery of interesting and novel mathematical theorems. Consider, for example, Euclid's proof that the interior angles of a triangle add up to 180°. Euclid draws a representative triangle – an arbitrary three-sided plane figure ABC (see Figure 2). Next, he extends the base of this triangle from C to D and draws a line CE parallel to side AB. He then argues (by appeal to previously established theorems) that the three angles created by these new lines are equal to the interior angles of the triangle. And since these three angles form a straight line, they add up to two right angles, or, 180°.

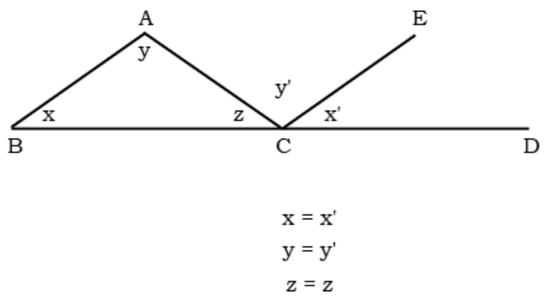

Figure 2

As Peirce notes, the procedure of Euclid's proof cannot be arrived at merely by analysing the meaning of the terms of his problem and applying the postulates and axioms of Euclidean geometry. The additions to the initial diagram he makes are not deduced from information given at the outset. As Peirce sees it, they are conjectures concerning the relevant procedures for investigating certain properties of triangles arrived at non-deductively through creative insight – what Peirce calls 'abduction'. Adding the lines CD and CE to the initial diagram is thus akin to carrying out an experimental procedure guided by a hypothesis in physics. A certain modification is made to the diagram and the consequences resulting from this modification are observed. It is then concluded (by induction) that the observed results would occur whenever the same operations are performed on a diagram of the same sort. Thus, even though Euclid's proof is deductive – his theorem follows necessarily from previously established propositions and axioms by legitimate inference rules – and does not rely on any knowledge of the actual world, it remains, for Peirce, an experimental result.

3. The Certainty of Mathematical Knowledge

On Peirce's view, results in both mathematics and the natural sciences are confirmed by observation using one and the same experimental method. Moreover, they are true in precisely the same sense – they are conclusions to which all rational inquirers would be led were inquiry rightly conducted and pursued sufficiently far. But if the findings of mathematics and the natural sciences are on the same epistemological footing, how can Peirce claim that mathematical results offer any more secure a foundation for logic than results in the natural sciences do?

Peirce's answer is that the very nature of diagrams is such that certain kinds of error that occur in the natural sciences cannot arise in mathematical inquiry. Unlike mathematicians, natural scientists are not interested in formal relations among symbols *per se* but rather in relations among objects in the world that symbols purport to represent. By reasoning with the symbols through which they express their theory of the world, natural scientists draw conclusions about objects in nature. But even when these conclusions are validly derived there remains the possibility that they fail to represent the way things actually are. There is no guarantee that the structure of the symbols devised to understand nature corresponds to the structure of objects in the natural world.

On Peirce's view, however, there can be no analogous discrepancy between the conclusions mathematicians arrive at by reasoning with symbols and the mathematical facts they are out to investigate. The subject-matter of mathematics *is* the formal structure of diagrams. In the terminology of Peirce's theory of signs, a diagram is an 'icon'. It thus represents a formal structure by exemplifying or replicating it (CP 1.369; 1.558 and 2.276). A diagram cannot misrepresent its formal structure since the configuration of its elements makes it the diagram it is. In the Euler diagram examined earlier, the configuarion of circles in the diagram instantiates the very relations of class inclusion that the mathematician is out to study. Similarly, in Euclid's proof that the interior angles of a triangle add up to 180° the spatial relations among the elements of his diagram are the very geometrical relations under study. Mathematical diagrams cannot fail to represent mathematical objects faithfully because the formal relations exemplified by the elements of these diagrams are the mathematical objects.

To better see Peirce's point, consider the difference between studying the structure of symbols on a map to determine what it says and studying the map to determine the configuration of objects in the world it purports to represent (4.530). While a map can be misinterpreted, it cannot misrepresent its own content because that content is determined by the configuration of marks that constitute the map. As Peirce sees it, map reading is akin to the study of diagrams in mathematics – the relations among the symbols are the mathematical objects to be studied and a diagram cannot fail to exemplify the symbolic relations that constitute it.

Navigating with a map is a different story, however. In this case the aim is to use the map as a sign of things in the world beyond it. Even a correctly interpreted map will mislead if the structure of its symbols does not correspond to the configuration of objects in nature. For Peirce, using a map to navigate in the world is like using a theory in the natural sciences to understand the world beyond the theory. Since the structure of reality cannot be discerned merely by discovering relations among symbols, the natural sciences are subject to errors in ways mathematics is not.

Peirce's point is not that mathematics differs from the natural sciences in being immune from error. A mathematician may go wrong by reading a diagram incorrectly or performing operations on it incorrectly, just as a natural scientist might make an observational or procedural error in the laboratory. His point is rather that a diagram is what it is by virtue of the way its elements are configured and the way its elements are configured is precisely what the mathematician is out to discover. Unlike symbols used in the natural sciences which may be configured in ways that are at odds with the objects in nature they are intended to represent, the structure of mathematical diagrams is inherently veridical and cannot fail to disclose the nature of the objects they purport to represent.

It is also important to notice that in claiming that mathematical inquiry is more secure than inquiry in the natural sciences Peirce is not contradicting his view that all scientific knowledge stands on the same epistemological footing. As noted already, one and the same experimental method is used in mathematics and the natural sciences and in both cases conclusions are true or false in precisely the same sense. Any difference in the degree of certainty in their results arises from the nature of their respective subject-matters not from any difference in their epistemological underpinnings.

It might be thought that mathematical inquiry is different methodologically from inquiry in the natural sciences inasmuch as results in physics, say, require the examination of large samples of objects and the performance of numerous experiments on them. In mathematics, by contrast, a single diagram might be supplied to establish a general result (as in the case of Euclid, for example) and worries about sample size and repeated trials seem not to arise.[7] How are these differences to be accounted for if, as Peirce claims, inquiry in mathematics and the natural sciences are warranted on precisely the same grounds?

To Peirce's way of thinking this objection overlooks the lengths mathematicians go to to ensure their findings are replicable – their calculations are double-checked and their proofs subjected to the critical scrutiny of others. While he admits that errors are on the whole easier to detect in mathematics than in the natural sciences, Peirce thinks this is because experiments in mathematics are performed on diagrams and do not require the construction of the sort of specialized laboratories used in physics and chemistry or the sort of extensive fieldwork required in biology or anthropology. Experiments on diagrams "can be multiplied *ad libitum* at no more cost than a summons before the imagination" (CP 4.531) and so mathematicians can "glut [themselves] with experiments" to guard against procedural slips, misperceptions and hasty generalizations (CP 4.87). Natural scientists, by contrast, find it "most troublesome to obtain any [experiments] that are satisfactory" (*Ibid.*).

Moreover, Peirce thinks spurious generalizations are less apt to arise in mathematics because the objects under study are constructed by mathematicians, rather than found in nature as in the natural sciences. Euclid's diagram is deliberately designed to represent the formal properties of a triangle and an Euler diagram is explicitly constructed to show the relevant formal relations among terms. So long as these constructions are carefully done and properly observed, conclusions concerning their formal relations will necessarily hold of any diagrams having the same form. The circumstances of mathematical inquiry are thus far more conducive to discovering reliable generalizations than are the cirrcumstances of the natural scientist who discovers, rather than constructs, her objects of study and must sort out which among their properties admit of reliable generalization and under what conditions.[8]

Still, for Peirce, the various differences between mathematics and the natural sciences derive from differences in their respective subject-matters, not their methods or epistemological credentials. Generalizing experimental results in mathematics can still go awry in precisely the same way it does in the natural sciences. Just as a physicist might draw a hasty conclusion about the behaviour of light from a sample of data that is not random or sufficiently large, so too a mathematician might draw a hasty conclusion from a diagram – for example, by concluding from experiments performed on an equilateral triangle that all triangles have equal angles. Euclid avoids this sort of error by performing his experiment on an arbitrary – or randomly selected – triangle but even he fails to recognize that his conclusion does not generalize to curved space.

Finally, it might be urged against Peirce's view that mathematical conclusions being deductive and formal are tautologies and devoid of factual content. The suggestion is that since claims in mathematics (and claims in logic that depend on them) deal only with relations among symbols and are true regardless of the way objects in nature are configured, they are not factual and thus of a different epistemological kind than claims in the natural sciences.

Peirce sees no basis for any such distinction, however. For him, inquiry, whether in mathematics or in the natural sciences, has a single aim – namely, to discover true propositions by means of the scientific method. Moreover, for him, the very notion of a true proposition implies an accurate representation of something real. Triangles have a definite structure that rational inquirers can get right or wrong, no less than white light does. This structure is instantiated in Euclid's diagram and objectively uncovered through the experimental procedures that culiminate in his proof, just as properties of white light are disclosed in Newton's *Experimentum Crucis*. Peirce agrees that, unlike Newton's findings, Euclid's conclusion that the interior angles of a triangle total 180° is formal, if by this it is meant that it holds whether or not triangles actually exist. Nevertheless, he insists, its truth or falsehood is independent of what any inquirer believes and thus remains a matter of fact. He also acknowledges that knowledge of nature cannot be derived from Euclid's conclusion (or from any other mathematical result) but insists it means only that mathematics deals with conceivable possibilities, that is, with the way things might be rather than with the way they actually are. He does not think it follows from this that mathematical propositions are without content or that they are true in a way that is different from the propositions of the natural sciences.

4. Conclusion

For Peirce, the project of providing a philosophical foundation for science firmer than science itself is incoherent. There is, he thinks, no knowledge that is prior to, more secure than, or different in kind from, empirical inquiry. But he does not see this as a reason to abandon the view that mathematics and logic are autonomous from the natural sciences. For him, results in mathematics and logic are discovered through application of the scientific method. However, logic is founded on mathematics and mathematics uncovers truths about the structure of diagrams that are justified without appeal to findings in the natural sciences and remain true whatever inquiry in the natural sciences might disclose to be the case.

Peirce's distinction between mathematics and logic on the one hand and the natural sciences on the other harkens back to the traditional distinction between knowledge rooted in relations among ideas (or in Peirce's case, among signs) and knowledge rooted in relations among facts. However, on his rendering this distinction is drawn within the subject-matter of science so as to remain compatible with his insistance that both sorts of knowledge are justified by the experimental method. Although logic and mathematics are independent of the science of the actual world, they offer no counterexamples to the claim that all knowledge is derived from experience.

Peirce thinks knowledge attained in mathematics and logic differs from that disclosed by the natural sciences in being purely formal. Results in mathematics and logic are derived from examination of the relations among signs rather than the objects those signs represent and do not reveal knowledge of the actual world. Yet, for him, this does not imply that they are empty of factual content or fail to represent states of affairs. Mathematical and logical truths are true and true propositions represent reality, just as findings in the natural sciences do.

Finally, Peirce takes knowledge in every science to be fallible. There is no guaranteeing that any scientific result is true and immune from future revision. Yet he sees this as compatible with the view that findings in mathematics and logic are, in virtue of their subject-matter, capable of greater certainty than those in the natural sciences. As a result, he takes mathematics to provide a more stable foundation for the theory of inquiry than results in the natural sciences can. For him, mathematics, and by extension logic, retain something of their foundational role in scientific philosophy, even after the quest for certainty has been shown to be illusory.

I have tried to show that Peirce's theory of icons – specifically his account of diagrammatic reasoning – lies at the heart of his account of knowledge in mathematics and logic. Since mathematics and logic are the foundation of Peirce's epistemology and metaphysics, his theory of icons is crucial to his entire philosophical system. The account of mathematics and logic Peirce derives from his account of icons is original and deserves far more attention than it has received to date.[9]

Notes

1. I follow standard practice in citing passages from Peirce's works. References to the Harvard edition of Peirce's *Collected Papers* (Peirce 1931-1958) give the volume and paragraph number (*e.g.* CP 3.452 refers to volume 3, paragraph 452). References to the chronological edition of Peirce's *Writings* (Peirce 1982) are by volume and page number (*e.g.* W1 : 357 refers to volume 1, page 357) References to *The New Elements of Mathematics* (Peirce 1976) likewise cite the volume and page number (*e.g.* NEM 4 : 196 refers to volume 4, page 196).
2. As truth is a property of signs, Peirce takes logic to fall under the general theory of signs, or, semiotics (CP 1.539 and 2.227). For him, the aim of logic is to determine the conditions under which true signs are realized. Logic so construed extends beyond deduction to abduction (the logic of theory formation) and induction (the logic of confirmation) and it includes the theory of scientific method and of truth. On his view, logical laws (a) hold whatever else inquiry may disclose to be the case; (b) delimit meaningful possibilities but do not supply knowledge of actual states of affairs; (c) bind knowers in all times and places, regardless of the subject-matter; (d) must be true if knowledge is to be possible and (e) determine the general form of reality insofar as it can be represented cognitively. For detailed discussion see Forster (2011).
3. Peirce writes : "By the theory of cognition is usually meant an explanation of the possibility of knowledge drawn from principles of psychology. Now, the only sound psychology being a special science, which ought itself to be based upon a well-grounded logic, it is indeed a vicious circle to make logic rest upon a theory of cognition so understoood" (CP 3.432). See also W1 : 362 and CP 2.210.
4. Although phenomenology is fundamental to Peirce's philosophy, his views of the subject are not relevant to my purposes here.
5. Peirce argues against attempts to ground knowledge on intuition – that is, on knowledge that purports to be independent of any previous knowledge and arrived

at without reasoning from signs – in "Questions Concerning Certain Faculties Claimed for Man" (CP 5.213-5.263 or W2 : 193-211).

6. Newton further concludes that the prism does not create the spectrum but rather merely separates out the elements of white light. Controversies surrounding this conclusion have no bearing on my use of the example.
7. See, for example, Russell (1912/1997), especially Chapter VIII 'How *A Priori* Knowledge is Possible".
8. Martin Lefebvre suggests that on Peirce's view natural science is also less certain than mathematics because the natural world is governed by chance and the mathematical world is not. I agree but insist that, for Peirce, this is a purely contingent difference between mathematics and natural science. While Peirce thinks the laws of nature are in fact statistical, he also allows that this might not have been the case. Indeed, he thinks the world is progressing towards a limit in which there is strict determinism.
9. I am grateful to Andrew Lugg for his careful comments on numerous drafts of this paper. I am also indebted to Martin Lefebvre for his comments and for reading, on short notice, an earlier version of this paper on my behalf at the conference on 'Peirce and the Image' held at the International Centre for Semiotics and Linguistics at the University of Urbino in Urbino, Italy on July 17, 2006.

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Abstract

Peirce's insistence that logic and mathematics are prior to, and independent of, the natural sciences faces serious objections. First, his claim that all knowledge is scientific seems to imply that any justification of the principles of scientific method presupposes the legitimacy of those principles and thus is circular. Second, his claim that truths of logic and mathematics hold independently of facts about the actual world seems hard to square with his insistence that they are established by observation using the experimental method. Finally, his view that logic and mathematics are sciences on the same epistemological footing as any other science seems at odds with his view that results in mathematics and logic are more secure than those of the natural sciences. I argue that Peirce's answers to these objections rest on his theory of icons. If this is right, his theory of icons is central to his epistemology and issues in a view of mathematics and logic that is original and has been largely overlooked.

Résumé

L'insistance de Peirce à montrer l'antériorité et l'indépendance de la logique et des mathématiques sur les sciences naturelles rencontre de sérieuses objections :

l'affirmation, en premier lieu, selon laquelle tout savoir est scientifique semble laisser entendre que toute justification des principes de la méthode scientifique présuppose la légitimité de ces principes, s'avère un cercle vicieux; sa prétention, en second lieu, à affirmer que les vérités de la logique et des mathématiques se tiennent en dépit des faits concernant le monde actuel, semble difficile à cadrer avec l'insistance qu'il met à affirmer qu'elles sont établies par observation, suivant ainsi la méthode expérimentale; et, finalement, sa vision que voulant que logique et mathématiques soient des sciences au même titre epistémologique que toutes les autres, semble en contradiction avec sa vision voulant que les résultats, en mathématiques et en logique, sont plus fiables que ceux des sciences naturelles. J'argue que les réponses de Peirce à ces objections reposent sur sa théorie des icônes. Si cela s'avère, cette théorie est au coeur de son épistémologie et résulte en une vue originale des mathématiques et de la logique qui a été largement négligée

PAUL FORSTER has published numerous articles on the history of analytic philosophy, pragmatism past and present and Peirce's philosophy. He is a contributor to, and co-editor of : *The Rule of Reason : The Philosophy of Charles Sanders Peirce* (University of Toronto Press 1997) and author of *Peirce and the Threat of Nominalism* (Cambridge University Press 2011).