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Objective Reconstructions of the Late Wisconsinan Laurentide Ice Sheet and the Significance of Deformable Beds

Reconstitutions objectives de la calotte glaciaire laurentidienne au Wisconsinien supérieur et l'importance des lits non résistants

Objektive Rekonstruktionen der Eiskalotte des späten Wisconsin und die Bedeutung der nachgiebigen Betten

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Article abstract

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OBJECTIVE RECONSTRUCTIONS OF THE LATE WISCONSINAN LAURENTIDE ICE SHEET AND THE SIGNIFICANCE OF DEFORMABLE BEDS

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ABSTRACT A three dimensional steady state plastic ice model; the present surface topography (on a 50 km grid); a recent concensus of the Late Wisconsinan maximum margin (PREST, 1984); and a simple map of ice yield stress are used to model the Laurentide Ice Sheet. A multi-domed, asymmetric reconstruction is computed without prior assumptions about flow lines. The effects of possible deforming beds are modelled by using the very low yield stress values suggested by MATHEWS (1974). Because of low yield stress (deforming beds) the model generates thin ice on the Prairies, Great Lakes area and, in one case, over Hudson Bay. Introduction of low yield stress (deformable) regions also produces low surface slopes and abrupt ice flow direction changes. In certain circumstances large ice streams are generated along the boundaries between normal yield stress (non-deformable beds) and low yield stress ice (deformable beds). Computer models are discussed in reference to the geologically-based reconstructions of SHILTS (1980) and DYKE *et al.* (1982).

RÉSUMÉ *Reconstitutions objectives de la calotte glaciaire laurentidienne au Wisconsinien supérieur et l'importance des lits non résistants*. À partir d'un modèle théorique tridimensionnel de plasticité de la glace, de la topographie actuelle (sur un canevas de 50 km²), du nouveau consensus quant à la limite maximale de la marge glaciaire (PREST, 1984) et d'une carte des seuils de plasticité de la glace, les auteurs ont élaboré des modèles de la calotte glaciaire laurentidienne. On a donc reconstitué par ordinateur une calotte asymétrique à dômes multiples, sans idée préconçue quant aux directions de l'écoulement des glaces. On a évalué les conséquences de la présence éventuelle de lits non résistants en se fondant sur les très bas seuils de plasticité de la glace proposés par MATHEWS (1974). En raison des bas seuils de plasticité (lits non résistants), les modèles démontrent qu'une glace peu épaisse couvrait les Prairies et la région des Grands Lacs, ainsi que la baie d'Hudson, dans un des deux cas. La prise en considération de régions à bas seuils de plasticité (lits non résistants) montre également la présence de pentes faibles et des changements brusques de direction de l'écoulement glaciaire. Dans certains cas, de grands courants glaciaires se manifestent le long des limites entre les endroits où les seuils de plasticité sont normaux (lits rigides) et les endroits où les seuils de plasticité sont bas (lits non résistants). Les modèles obtenus par ordinateur sont ensuite comparés aux reconstitutions de SHILTS (1980) et de DYKE *et al.* (1982), élaborées à partir des données géologiques.

ZUSAMMENFASSUNG *Objektive Rekonstruktionen der Eiskalotte des späten Wisconsin und die Bedeutung der nachgiebigen Betten*. Ein dreidimensionales konstantes Modell der Eis-Plastizität, die gegenwärtige Oberflächentopographie (auf einem Gitternetz von 50 km), ein neuer Konsensus über den maximalen glazialen Rand des späten Wisconsin (PREST, 1984) und eine einfache Karte der Eis-Plastizitäts-Schwelle werden benutzt, um ein Modell der Laurentischen Eisdecke herzustellen. Eine vielfach gewölbte, asymmetrische Rekonstruktion ist hergestellt worden, ohne vorgefaßte Meinung über die Richtung des Fließens. Die Wirkungen von möglicherweise vorhandenen nachgiebigen Betten werden mittels der sehr niedrigen Eisplastizitätsschwelle, wie sie von MATHEWS (1974) vorgeschlagen wird, im Modell gestaltet. Wegen der niedrigen Plastizitätsschwelle (nachgiebige Betten) zeigt das Modell dünnes Eis in den Prairies, der Gegend der großen Seen und in einem Fall über der Hudson Bay. Die Berücksichtigung von Gebieten mit niedriger Plastizitätsschwelle (nachgiebige Betten) führt auch zu niedrigen Hängen und abrupten Wechseln in der Richtung des Eisfließens. Unter gewissen Bedingungen bilden sich breite Eisströme entlang der Grenzen zwischen Gegenden mit normaler Eisplastizität (beständige Betten) und geringer Eisplastizität (nachgiebige Betten). Durch Computer erstellte Modelle werden in Bezug auf die geologisch erarbeiteten Rekonstruktionen von SHILTS (1980) und *DYKE et al.* (1982) diskutiert.

INTRODUCTION

Why another ice sheet reconstruction

Andrews recently reviewed Late Wisconsinan ice sheet reconstructions (ANDREWS, 1982) and noted that many theoretically-based reconstructions tend to generate symmetrical ice cover with thick ice cover over the Canadian prairies and Hudson Bay (PATERSON, 1972; SUGDEN 1977; DENTON and HUGHES, 1981; BOULTON *et al.*, 1985; HUGHES 1985). In contrast PELTIER's (1981) reconstruction based on uplift data suggests generally thinner ice and very thin ice cover over the prairies. Recently REEH *et al.* (1983) and BOULTON *et al.* (1985) have introduced deformable beds into the ice flow modelling exercise. These beds have the effect of producing areas of thinner ice and removing much of the symmetry in the reconstructions. The Denton and Hughes model can also introduce deformable beds but they have not yet done much with the concept. Andrews also notes that some models rest sensitively on unknown input paramaters such as accumulation patterns at 18 ka BP and, yet others, by using some of the geological evidence to infer major flow lines, leave themselves with no independent evidence to check the results of the model.

We present here an expansion of the (REEH *et al.*, 1983) computer reconstructions of the Late Wisconsinan ice cover. We use a very simple ideal plastic ice rheology that is rather insensitive to unknown parameters and takes as input only the margins of the ice sheet, the present day topography, and an assumed yield shear stress, τ_o. A flow line calculation begins at a point on the margin and integration of the equation proceeds up slope using only the margin's shape, the underlying topography and the yield stress. The surface elevations and trajectories are calculated for many flow lines around the entire margin. *No* assumptions are made in advance about ice divides, centres or streams; these fall objectively out of the calculations. There are weak assumptions made about accumulation rates and ice temperatures because these influence τ_o very weakly. REEH (1982) developed and used an earlier version of the program on the Greenland ice cap, successfully reproducing all the major ice divides, ice streams and centres. His calculated flow line trajectories were accurate and the model's surface elevations were at most in error by 10%.

Also we attempt to allow for the effects of deformable beds in the western plains, Hudson Bay and the Great Lakes regions of the ice sheets. Deformable beds are modelled here by simply using small values of yield stress τ_o for the ice. The computed ice sheets are non-symmetric with thin ice over the prairies, thick ice on the Canadian Shield, and in the case of low τ_o Hudson Bay beds with thin ice in Hudson Bay. The predicted domes, ice divides and ice streams can be favourably compared to much of the surface glacial morphology.

We think that the model provides a robust, objective, and economical tool for linking and cross-checking the two main types of geomorphological field evidence, namely ice margins and flow direction indicators (*i.e.* striae, erratic trains, etc.).

THE MODEL

The model is detailed by REEH (1982) and the following summarizes it. Ice is taken as an ideal plastic with a yield stress τ_o normally in the range 0.5 to 1.5 bars (1 bar $= 10^5$N m^{-2}). Neglecting longitudinal stress gradients and transverse shear stress, the ice elevation E as function of distance S measured positive up a flowline is governed by:

$$\frac{dE}{dS} = \frac{H_f}{H} \tag{1}$$

with $H_f = \tau_o/\rho g$ (H_f has a dimension of length.)
where ρ is the ice density (920 kg m^{-3})
g acceleration due to gravity (9.81 m S^{-2})
H the ice thickness (m)
τ_o the yield stress is in N m^{-2}

In the case of a two-dimensional ice sheet on a horizontal bed, equation 1 integrates to the form frequently used by glacial geomorphologists to estimate ice thickness at ice divides

$$H_o^2 = \frac{2\tau_o}{g\rho} L \tag{2}$$

with H_o the maximum thickness at the divide
L the distance between the divide and the margin.

If the accumulation rate, a (m ice per year) and basal temperature T (°C) are constant, the equivalent plastic yield stress can be estimated by (REEH, 1982).

$$\tau_o = \left(\frac{(n+2)\, ga\rho}{4B_o \exp (kT)} \right)^{1/(n+1)} \tag{3}$$

The quantities n, B_o and k come from Glen's Flow Law for ice and are 3, 0.5 bar^{-3} year^{-1}, and 0.2 deg^{-1} respectively (REEH, 1982).

Equation 3 is not used to calculate the τ_o's but serves as a guide in assessing how to vary τ_o when accumulation and temperature T change. τ_o is seen to be rather insensitive to a and T. In the program τ_o is an average value for an entire flow line, unless it crosses the heavy hatched lines of Figure 1.

The computational algorithm comes from the assumption that equation 1 is true along any flow line and further that

$$\overrightarrow{\nabla E} \times \overrightarrow{ds} = 0 \tag{4}$$

Equation 4 states that vector increments $\overrightarrow{ds}$ along a flow line are perpendicular to the surface slope. Equations 1 and 4 can be solved by the method of characteristics for arbitrary margins and bed topographies. The original development by REEH (1982) must be extended slightly here (see Appendix) to include terms that become large when the yield stress τ_o changes significantly along a flow line, *e.g.* when a flow line crosses from (deformable to non-deformable beds) normal to low τ_o ice.

The solution allows the bed to be depressed isostatically to B_d (relative to present sea level) using

$$B_d = B_a - H \frac{\rho}{\rho_r} \qquad (5)$$

where B_a is the present 'unloaded' elevation
ρ_r is the density of the upper mantle (3300 kg m^{-3})

Since time does not enter what is basically a force balance solution, the isostatic adjustment has no time delay.

THE MARGIN MAP

The time instant we attempt to reconstruct is nominally the 18 ka extent of the Laurentide Ice Sheet. A great deal of uncertainty still exists about the northern and northeastern positions of this margin (ANDREWS, 1982). Probably this maximum margin was not synchronous. The age spread around 18 ka for maximum margins seems to be $\pm$ 4 ka (Prest, personal communication).

We chose a slightly smoothed version of one of two proposed alternatives for the Late Wisconsinan. PREST (1984) refers to them as the minimum and maximum concepts. Even if the entire margin is not a synchronous feature the reconstruction is valid for local areas scaled by lengths of margin that are known to be synchronous. We present the minimum concept results here. The maximum concept results with deformable Prairie and Great Lakes beds have been presented in a poster session (REEH *et al.*, 1983) (see cover).

BED TOPOGRAPHY MAP

A one-square degree data base (USDMAAC, 1976) is used to generate a regular 50 by 50 km grid of the crusts' surface elevations. The grid is with respect to a cartesian coordinate system laid over a Lambert conformal conic projection with two standard parallels at latitudes 49° and 77°. Linear distortions are less than 3%. Where the ice margins are beyond the present shore one should unload the undersea topography before depressing it, but since the depths involved are small we neglect this effect. All interpolations of surface topography are done using spline functions. No attempt is made to allow for any residual rebound left in the present topography. This is largely because there is no agreement on the magnitude or response time of the long-term mantle readjustment and, even if there was, we would not know the phase of the readjustment at 18 ka BP because that would require a loading history stretching back over 100 ka.

THE YIELD STRESS MAP

Ideally the equivalent plastic yield stress map would be represented with the same detail as the topography and vary with basal temperature accumulation and bed type. Since there is little hard data we chose a very simple map of τ_o (Fig. 1).

REEH (1982) used the model to generate accurate surface elevations, ice divides and ice streams of present-day Greenland. He obtained best results for flow lines in north and central Greenland with τ_o 0.65 and 0.9 bars respectively. These flow lines include along their lengths variable accumulation and basal temperatures, but the insensitivity of τ_o (see equation 3) allowed the use of a single average value for each region. The northern flow lines have a lower τ_o (in line with equation 3) largely because north Greenland has low accumulation, *i.e.* 0.15 m yr^{-1} (ice equivalent) as opposed to 0.3 for central Greenland.

Our τ_o assignment for coastal high accumulation regions and northern or inland moisture starved regions follows the Greenland calibration (REEH, 1982). The ice thicknesses depend on the choice of τ_o, but experiment shows that the flow directions are insensitive to τ_o in the normal range.

Over large areas in western and south central North America we take τ_o very much smaller than 'normal' because of the thin ice profiles suggested by field observations (MATHEWS, 1974) and theory (BOULTON and JONES, 1979). Small τ_o does not imply that the ice itself is abnormally soft, but more likely that the bed sediments are themselves deformable. We allow for this by using small τ_o's derived from Mathews' inferred surface profiles. Much of the boundary between low τ_o and normal τ_o is taken as the edge of the Canadian Shield and is shown in Figure 1a by a hatched line. The "normal" τ_o values (0.5 bars $< \tau_o <$ 1.5 bars) are used

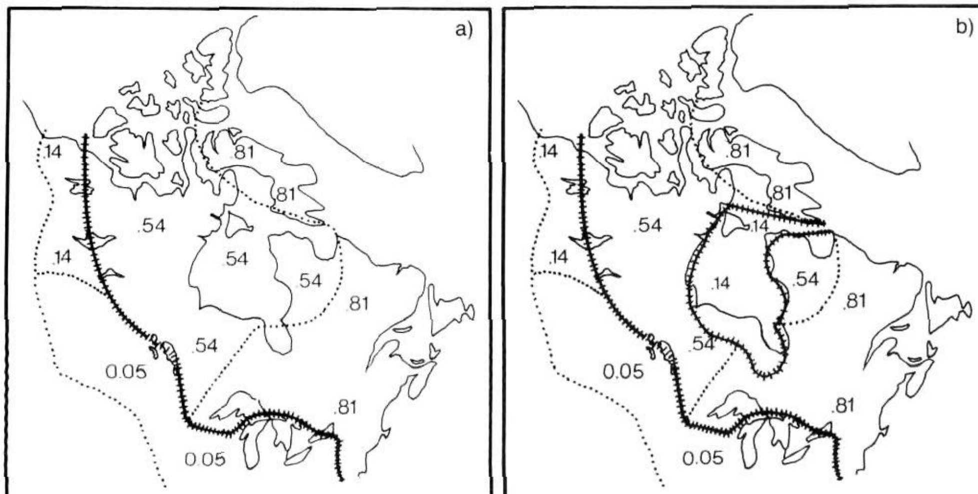

FIGURE 1. The effective yield stress map used in the reconstructions with: a) deformable beds only for the Prairies and Great Lakes regions; b) deformable beds also for the Hudson Bay area.
Carte de la plasticité de la glace qui a servi à l'élaboration des reconstitutions avec: a) la présence de lits non résistants dans les régions des Prairies et des Grands Lacs; b) la présence de lits non résistants dans la région de la baie d'Hudson.

over granite and plutonic rocks and the low τ_o values ($\tau_o <$ 0.2 bars) are confined to Paleozoic rocks, Proterozoic sediments and unconsolidated sediments. Two complete reconstructions were done, one with 'normal' yield stresses for the Hudson Bay — Hudson Strait area and one with $\tau_o = 0.14$ bars. MATHEWS' (1974) analysis of ice age nunataks in the Prairies allowed empirical estimation of τ_o on the Prairies and BOULTON and JONES (1979) give some justification for the low τ_o values in the Great Lakes region. Thus, Figure 1a has some empirical validity.

There is no similar direct evidence that the Paleozoic and Proterozoic beds under and around Hudson Bay were deformable. Although these sedimentary rocks are rather weak and relatively easy to break up (Shilts, personal communication), it is hard to imagine that they are deformable in the same sense as the thick, loose drift of the North American Plains. Also the Hudson Bay Lowland glacial sediment cover, though thick, does not show signs of deep disturbance (Shilts, personal communication) as would be expected with the prairie type of deforming beds. With these reservations in mind we try the experiment of assigning relatively low values of τ_o (.14 bars) to Hudson Bay and the Lowlands. BOULTON *et al.* (1985) feel the same way but do not go into details as to how or why they obtain the equivalent of our low τ_o. Figure 1a shows the τ_o map used with "normal" τ_o values for Hudson Bay and Figure 1b with small values for the Hudson Bay region. Given that water pressure is a key factor in the deformation of beds (BOULTON and JONES, 1979), the ice's basal temperature must be at the pressure melting point. If such beds are frozen they will not easily deform and the appropriate τ_o would then be in the normal range. Thus, one could hypothesize Figure 1a as a frozen Hudson bed case and Figure 1b as a melted Hudson bed case.

Basal temperatures under large ice masses are dependent on many factors (PATERSON and CLARKE, 1978) such as surface air temperature, surface melt, thickness, accumulation or ablation rate, ice velocity, ice deformation heating, and geothermal heat flux. For basal temperatures of thick ice there is a long-time delay (a few thousand years) between changes in surface variables and changes at the bottom. Later we will come back to the possibility that the Hudson Bay area's basal temperature went through episodes at and below the melting point with the beds consequently alternating between deformable and hard states. Finally, we assume that ice rheology was like present Holocene ice though there is some evidence that Wisconsinan ice laden with high microparticle concentrations is "softer" (PATERSON, 1977, 1981; FISHER and KOERNER, submitted; DAHL-JENSEN, 1985).

THE RECONSTRUCTIONS

Figure 2 presents the reconstruction using a hard or frozen Hudson Bay and Figure 3 using a low τ_o, soft Hudson Bay. Both assume very low yield stresses for the Prairies and Great Lakes regions. In both versions there are several identifiable ice centres and ridges. There is no trace of a single large ice centre, but rather multiple centres and ridges. The ice divides are heavy lines, elevations at 400 m intervals are medium solid lines, and the boundary between deformable and hard beds is given by dotted lines. Figure 2 shows the ice sheet consisting of three connected centres with a number of lateral spur ridges. Figure 3 has four major centres arranged in a U pattern around Hudson Bay. The maximum concept reconstruction (see cover), which assumed a hard Hudson Bay bed, had the same centres and major lateral ridges as the minimum concept reconstruction of Figure 2. The Laurentide ice volumes calculated for the minimum and maximum reconstructions are 21.1×10^6 km^3 and 25.9×10^6 km^3 respectively, assuming a hard Hudson bed.

The low τ_o Hudson Bay reconstruction has all the same centres, etc. as the frozen bed version, only the centres are farther away from the Bay and about 400 m lower. The Bay itself is an "ice sink" instead of being a high ridge area as in the normal τ_o reconstruction. The hard bottomed Hudson ridge K-H of Figure 2 seems to "become" the two widely-separated ridges K-H and Q-U of Figure 3. The total ice volume calculated for the minimum concept low τ_o Hudson Bay bed reconstruction is 18.0×10^6 km^3. For comparison, two recently proposed reconstructions, based on geological evidence, are presented in Figures 4 and 5. Figure 4 is due to DYKE *et al.* (1982) and is in good agreement with our deformable Hudson Bed reconstruction (Fig. 3). Figures 5, from SHILTS (1980), differs from Dyke *et al.* and our reconstructions in one major respect. Using the fact that some erratics founds on the southwest side of Hudson Bay and well into Ontario and Manitoba originated on the east side of Hudson Bay, Shilts invokes stable long east-to-west flow lines across southern Hudson Bay during much of the Late Wisconsinan. Dyke *et al.*, on the other hand, explain the erratics by a re-entrainment hypothesis that moves these rocks westward across the Bay over several cycles of growth and decay of the ice sheet. The Hudson Dome of Figure 4 precludes such long flow lines in the Dyke *et al.* reconstruction. DYKE *et al.* (1982) hypothesis in principle is attractive to us but one of their main arguments against Shilt's long flow lines is (in light of possible deformable beds) not as strong as it was. DYKE *et al.* (1982) considered a 1600 km Shilt's flow line from the Labrador ice divide to Lake Winnipeg and used equation 2 (with "normal" values of τ_o) to calculate that the elevation at the Labrador divide (Fig. 5) was 6 km. Then they calculated the elevation of the divide to be only about 3 km using the eastward flow lines from the divide 500 km to the coast of Labrador. This lack of symmetry about the Labrador ice divide and the resulting discrepancy in calculated divide elevations is a telling argument against the long Shilt's flow lines if τ_o is "normal" for *both* east and west flowing ice. But, if the west flowing ice is mostly over deformable beds with $\tau_o = .14$ bars, and the east flowing ice mostly over hard beds $\tau_o = .54$ to .81 bars, then one would expect there would be an asymmetry in the length of the flow lines and the ratio of the lengths of west flowing to east flowing lines would be the inverse of the ratios of the effective yield stresses, *i.e.* 3.9 to 5.8. The actual ratio for the lengths of Shilt's flow lines (west/east) is about 3.2. Also, with deformable beds included, the inconsistency in the calculated elevation of the Labrador ice divide is resolved. However, while our model with a soft Hudson bed certainly generates a feature like the Labrador divide (Q-U in Fig. 3), it does not produce the 1600 km long westward flow lines to Lake Winnipeg. Possibly there was a major bridge of low yield stress bed material between Hudson Bay and the Prairies that we should have included or maybe the erratic movements and these long apparent flow lines are not the result of a long-lived steady state ice sheet configuration. This will be discussed later.

DISCUSSION

WHAT HAPPENS AT τ_o BOUNDARIES BETWEEN HARD AND SOFT BEDS?

In the Appendix we describe what happens in this model when a flow line crosses from a hard bed (modelled by $\tau_o \sim 1$) to a deformable bed (modelled by $\tau_o << 1$) region. The ice surface slope changes by a factor $(\tau_{soft}/\tau_{hard}) \sim 1/3$ to $1/10$

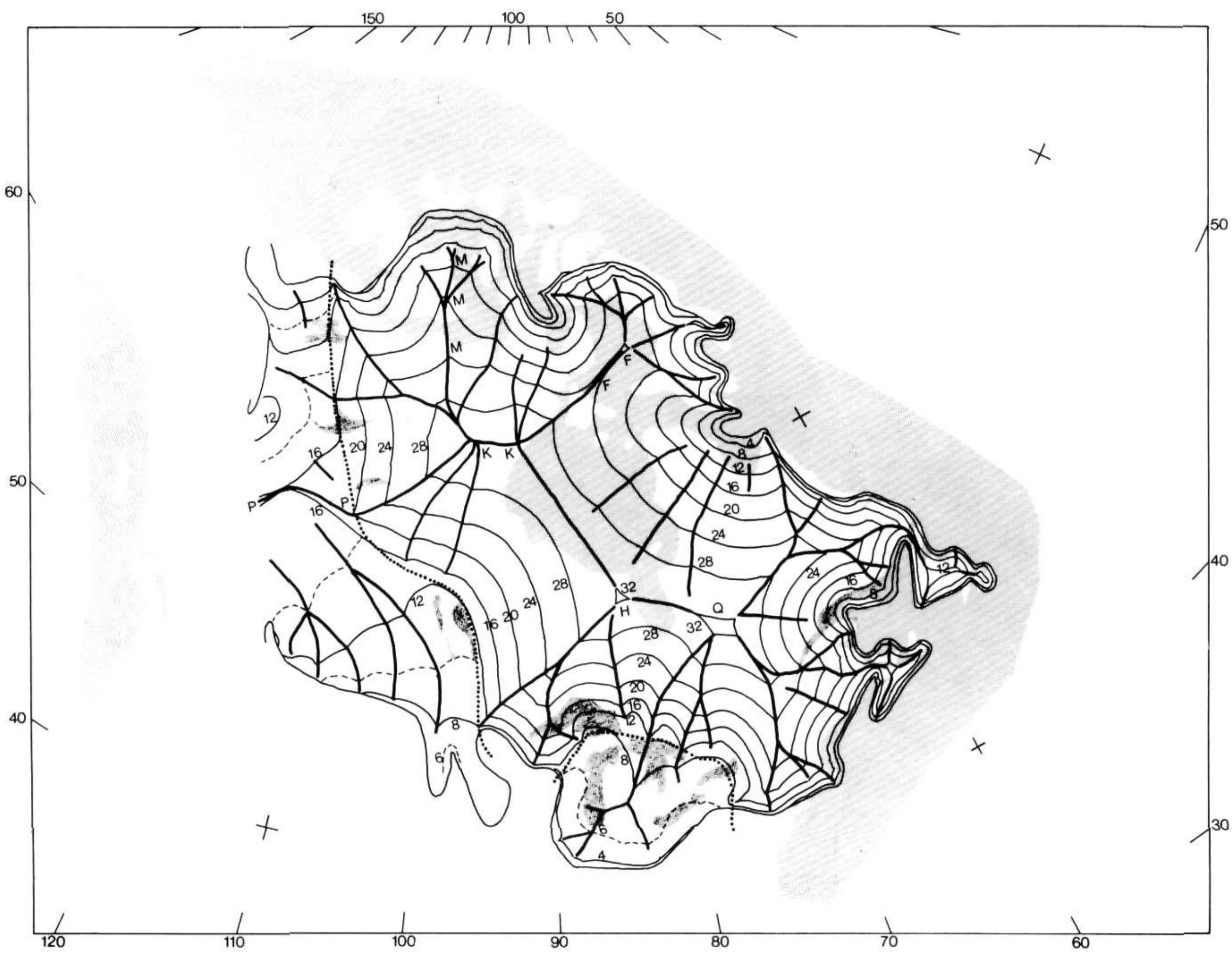

FIGURE 2. Computer reconstruction of the Laurentide Ice Sheet at 18 ka assuming a hard Hudson Bay and deformable Prairie and Great Lakes beds. Surface elevations are in 100's of metres above present sea level.
The main features lettered on Figures 2 and 3 are: H: Southern Hudson dome; Q: Québec dome-ridge, connected to H by a ridge; K: a Keewatin centre connected to H by a ridge; M: a long ridge covering M'Clintock Channel and joining K; Q-U: Québec-Ungava ridge in Figure 3; P: a low elevation ice divide in the low yield stress Prairies which separates North and South flowing ice; F: Foxe dome on or near Baffin Island. There are also local domes on Newfoundland, Nova Scotia and southern Baffin Island. Appalachian ice is largely local.

Reconstitution par ordinateur de la calotte glaciaire laurentidienne vers 18 ka en supposant la présence d'un lit rigide dans la région de la baie d'Hudson et de lits non résistants dans les régions des Prairies et des Grands Lacs. Les courbes de niveau sont en centaines de mètres au-dessus du niveau actuel de la mer.
Les principaux éléments illustrés sur les figures 2 et 3 sont les suivants: H: le dôme méridional d'Hudson; Q: le dôme allongé du Québec relié à H par une crête; K: le centre du Keewatin relié à H par une crête; M: une longue crête au-dessus du détroit de M'Clintock relié à K; Q-U: la crête du Québec-Ungava (fig. 3); P: une ligne de partage des glaces à basse altitude dans la région des Prairies où les seuils de plasticité sont bas et qui sépare les écoulements glaciaires vers le nord de ceux vers le sud; F: le dôme de Foxe sur l'île de Baffin ou à proximité. Il y a également des dômes locaux à Terre-Neuve, en Nouvelle-Écosse et dans la partie méridionale de l'Île de Baffin. La glace appalachienne est en grande partie d'origine locale.

and the direction of the flow lines can change dramatically. In low τ_o areas ice flow is very sensitive to topography and is more readily diverted to low areas or around uplands whereas, in hard bottom areas, the bottom topography is only partially controlling flow direction.

The deformable Prairie beds thus produce the relatively thin ice cover and the remarkable abrupt direction changes (Fig. 6) along the boundary between the soft Prairie sediments and hard Canadian Shield rocks. These model-generated results would seem to be in excellent agreement with flow directions on the western part of DYKE et *al.* (1982) map (Fig. 4) even to the correct placement of the Prairie ice divide labelled P in Figures 2, 3 and 4.

Given certain geometric relationships between the flow line directions, topography and the τ_o boundary, we found areas where there was a strong convergence or focussing of the flow lines. Figure 6 shows such a very large ice stream generated by the model. One can speculate that the large lakes (*e.g.* Lake Winnipeg) are caused by this ice stream. Similar remarkable ice streams are model-generated flowing north on the east side of a soft-bottomed Hudson Bay and on the south side of Foxe Dome flowing east (Fig. 3).

SPECULATIONS ABOUT THE RECONSTRUCTIONS AND THE LABRADOR ERRATICS

What follows is a highly speculative effort to reconcile our steady state reconstructions with Shilt's erratics. If the bed under Hudson Bay is frozen solid, and is thus not deformable, then the steady state model ice sheet would look like Figure 2, whereas a long enough interval of time with a melted-

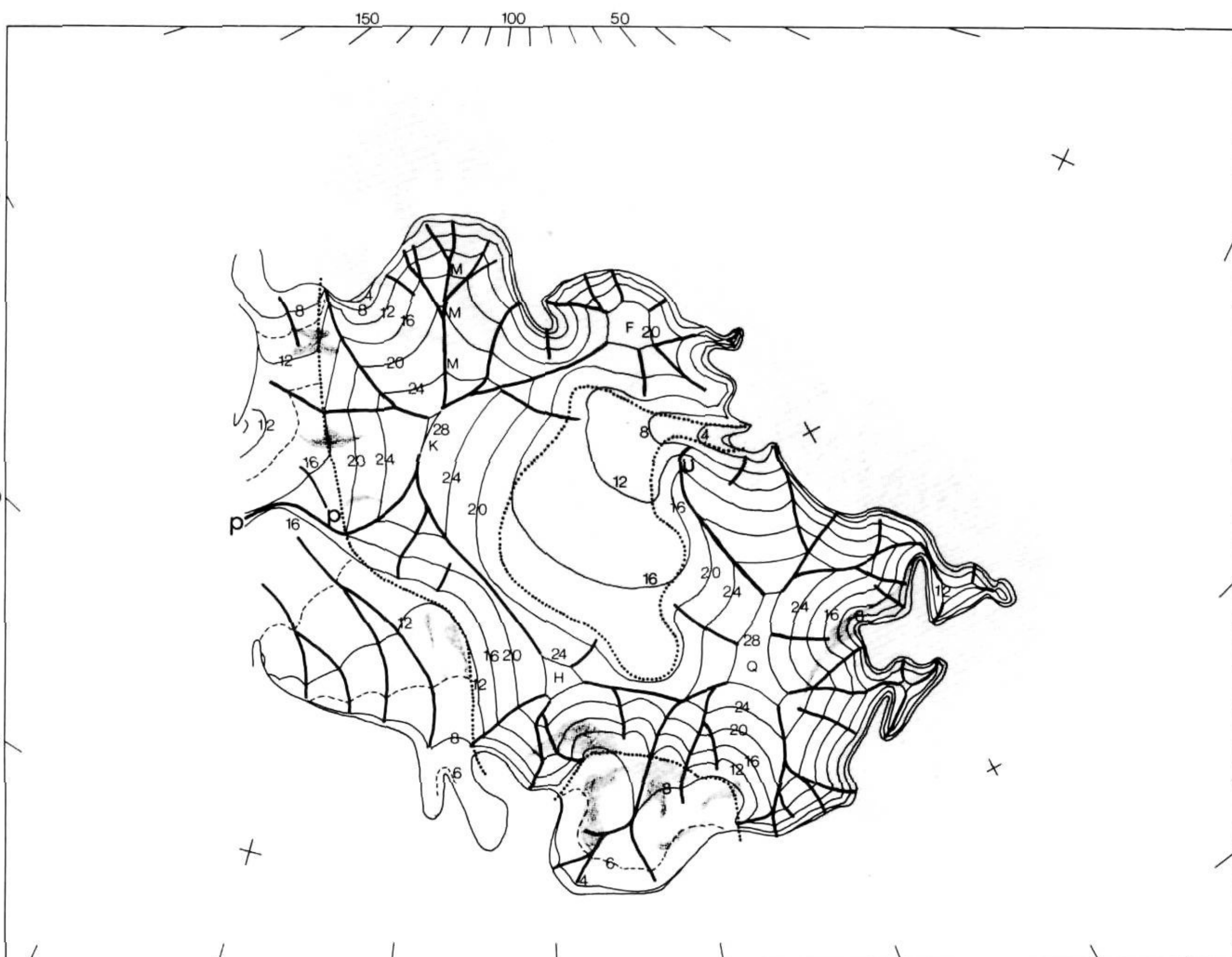

FIGURE 3. Computer reconstruction of the Laurentide Ice Sheet at 18 ka with deformable Hudson Bay, Prairie and Great Lakes beds. Surface elevations are in 100's of metres above present sea level. Lettered features are as in the caption of Figure 2. Ice flow lines are perpendicular to the elevation contours. Heavy lines denote ice divides and dotted lines mark the boundary between hard and deformable beds.

Reconstitution par ordinateur de la calotte glaciaire laurentidienne vers 18 ka en supposant la présence de lits non résistants dans les régions de la baie d'Hudson, des Prairies et des Grands Lacs. Les courbes de niveau sont en centaines de mètres au-dessus du niveau actuel de la mer. Les éléments identifiés par des lettres sont expliqués à la figure 2. Les directions de l'écoulement glaciaire sont perpendiculaires aux courbes de niveau. Les lignes grasses représentent les lignes de partage des glaces et les pointillés montrent la limite entre les régions à lits rigides et les régions à lits non résistants.

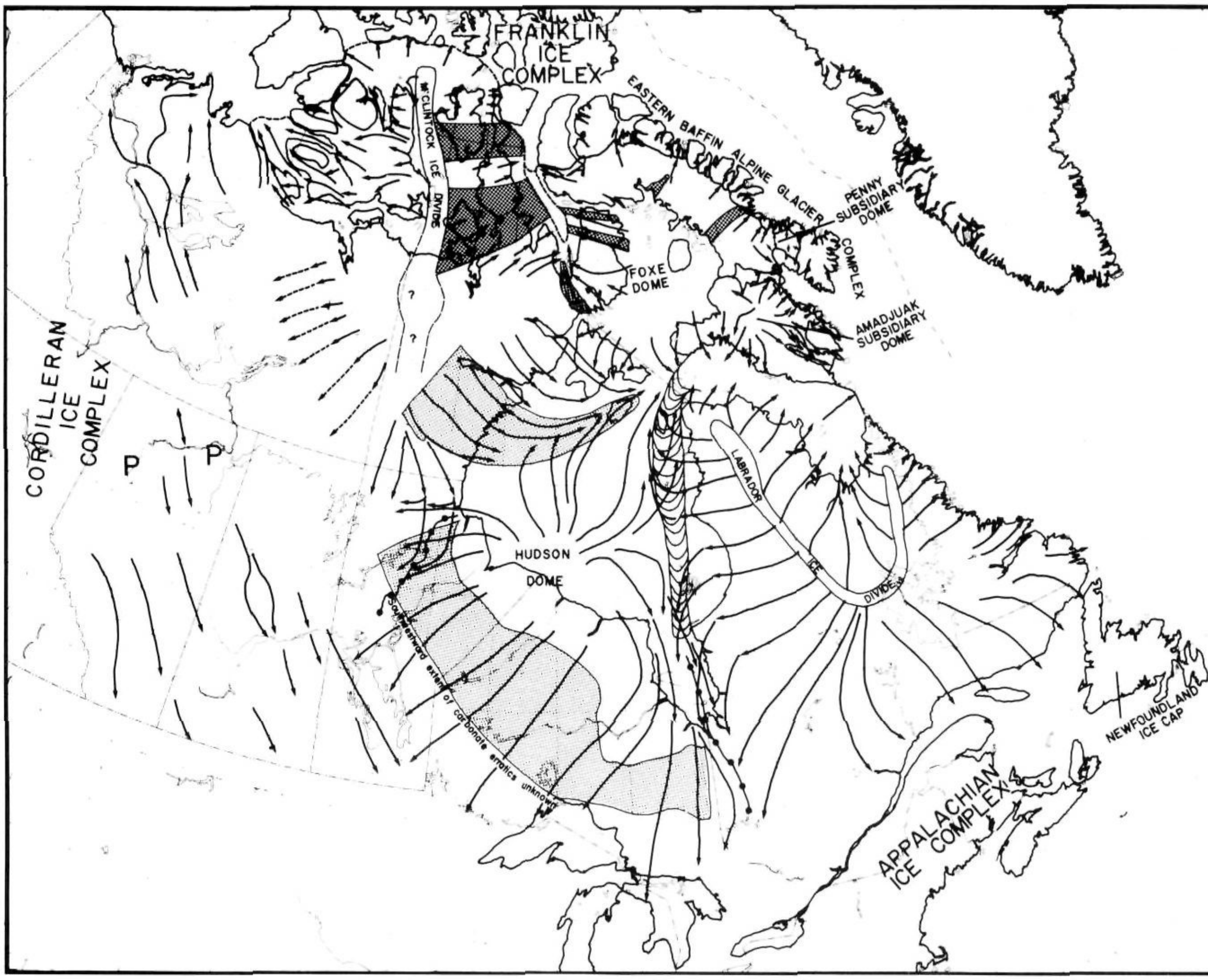

FIGURE 4. The structure and dynamics of the Laurentide Ice Sheet during the Late Wisconsinan maximum, from DYKE *et al.* (1982).

La structure et la dynamique de la calotte glaciaire laurentidienne au cours de la phase maximale du Wisconsinien supérieur selon DYKE et al. (1982).

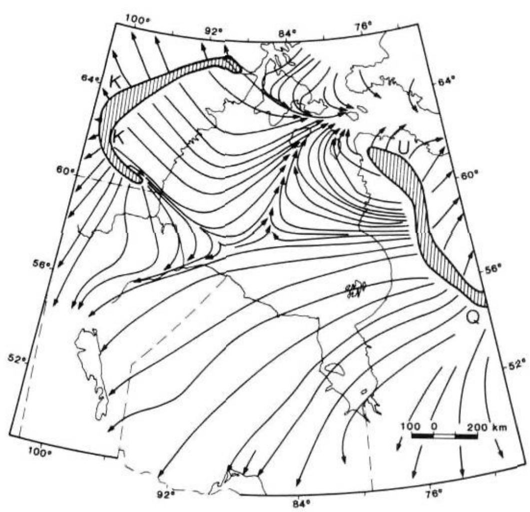

FIGURE 5. Flow pattern in the central part of the Laurentide Ice Sheet as inferred from erratics (SHILTS, 1980).

L'écoulement glaciaire dans la partie centrale de la calotte laurentidienne tel que suggéré par l'étude des blocs erratiques (SHILTS, 1980).

deformable Hudson Bay bed would produce a model steady state shown in Figure 3. One might consider these two possible states as extremes between which the real time varying ice sheet oscillated. If a case can be made for this hypothesis is then one could speculate that the Q-U Labrador ice divide of Figure 3 migrates from its asymmetric soft Hudson Bay position to the more symmetric central K-H position of Figure 2 which assumes a frozen or hard Hudson Bay bed. The migration of the Labrador ridge would sweep all the southern Hudson ice and its contents westward, and the position of the K-H ridge of Figure 2 would result in the continued westward movement of debris originating on the east side of the Bay.

There are some lines of evidence that suggest several cycles of growth and decay of the Hudson Bay ice during the Wisconsinan ice age. ANDREWS *et al.* (1983) maintain from amino acid analysis of shells that southern Hudson Bay was ice-free at 35 ka, 75 ka, and 105 ka; *i.e.* that there were three complete cycles of growth and decay of ice over Hudson Bay during the last glaciation. There are, however, some assumptions in the dating procedure, in particular one must know the whole temperature history of the samples to interpret the amino acid ratios.

BUDD and SMITH's (1981) "climate-ice sheet" model of the Wisconsinan glaciation could vary with time, and the main external forcing function, the total summer radiation over the last 200 ka years, generated three cycles of growth and decay. ANDREWS and MAHAFFY (1976) modelled the initial growth of the Wisconsinan ice sheet and found the Labrador centre grew from nothing westward right across Hudson Bay in only 7000 years. Certainly the Hudson Bay ice and, in fact, the entire ice sheet can disappear quickly (within a few thousand years), so there is time enough for several cycles during the last ice age.

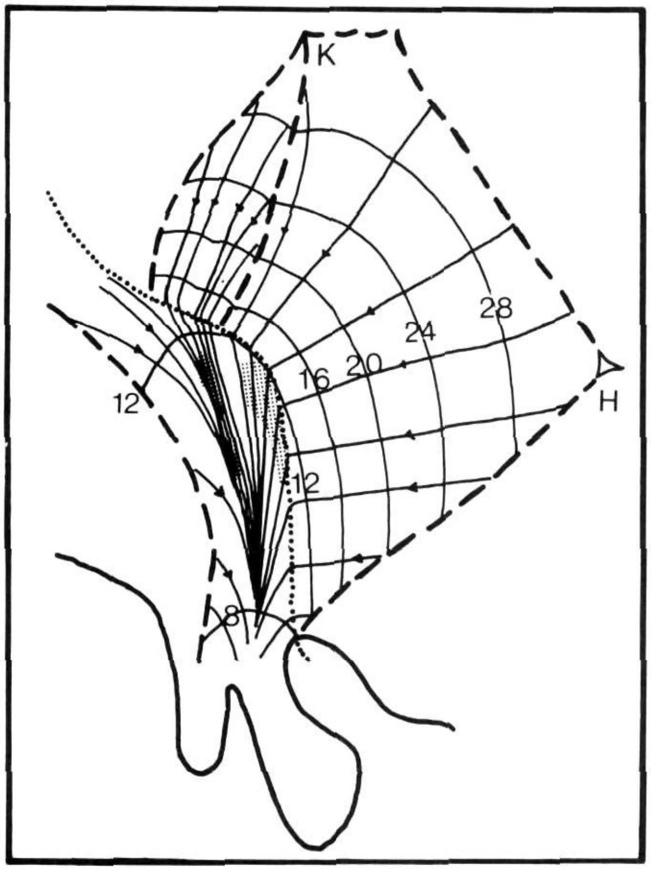

FIGURE 6. Reconstructed flow lines and surface elevations expanded from the Lake Winipeg region of Figure 2. As flowlines cross the dotted boundary line between hard Canadian Shield rock and deformable Prairie sediments they change direction toward the South and become highly concentrated into an ice stream. Also, the surface slope of the Prairie ice is only about 1/10th that of ice on the Canadian Shield. The heavy dashed lines denote ice divides and the heavy solid line is the margin. Similar ice streams are found on the east side of Hudson Bay and just south of Baffin Island in Figure 3.
Reconstitutions des directions de l'écoulement des glaces et extrapolation de l'altitude des surfaces à partir des données de la figure 2 sur la région du lac Winnipeg. Lorsque les glaces traversent les limites (en pointillés) entre les roches dures du Bouclier et les sédiments non résistants des Prairies, la direction de l'écoulement change vers le sud et les glaces se concentrent en un courant glaciaire. La pente de la glace qui couvre les Prairies ne représente que le dixième de celle du Bouclier. Le tireté gras montre les lignes de partage des glaces, tandis que la ligne grasse marque l'emplacement de la marge glaciaire. Des chenaux glaciaires semblables se trouvent dans la partie orientale de la baie d'Hudson et immédiatement au sud de l'île de Baffin (fig. 3).

The realistic calculation of basal temperatures of large ice sheets is beyond the scope of this paper, but since basal temperatures at the pressure melting point are needed to activate potentially deformable beds the topic is important. At this time not enough is known about the mechanical and thermal regimes of ice moving over active deformable beds to quantitatively evaluate the stability of the soft Hudson bed mode (Fig. 3). There is some guidance though for the other case of a frozen non-deformable Hudson bed (Fig. 2).

Near the central Hudson Bay ridge with the following plausible range of conditions, surface elevation 2800 m, thickness 4200 m ice surface air temperature $-35°C$, accumulation 0.20 to 1.0 m a^{-1}, yield stress $\tau_o \geq 0.7$ bars and geothermal heat flux 4.18×10^{-2} W m^{-2} and assuming a steady state condition with no horizontal advection terms, one would calculate using the simple heat transfer theory (PATERSON, 1981) that the basal ice was frozen and thus the bed non-deformable. However, CLARKE *et al.* (1977) have demonstrated that because of strain heating the solution of the steady state is multi-valued, and thus unstable under certain conditions. For example, under the above conditions, the solution is unstable and an initially-frozen bottom would warm up and melt within a few thousand years. One can conclude that there is a strong possibility that basal temperature under the hard Hudson Bay reconstruction would always tend to the melting point and thus set the stage for possible bed deformation and "collapse" to the soft bed case.

OTHER WORK AND FUTURE WORK

Recently BOULTON *et al.* (1985) have modelled the Laurentide Ice Sheet with and without deformable beds, and their latter reconstruction seems to accommodate the long flow lines of Shilts into a steady state model. The details of their reconstruction are a bit sketchy and their ice flow lines do not exhibit the direction changes one expects at the boundaries between hard and deformable beds. This is particularly apparent in the Prairie region of North America where their flow line directions differ from those inferred from the surface geology (DYKE *et al.* 1982) (Fig. 4).

In trying to reconcile all the glacial landforms with steady state models, we and others may be asking too much of the steady state. Even a sequence of steady states is no substitute for an explicit time variable. In addition to the climatic time variables of air temperature, accumulation, melt, equilibrium line elevation, and sea level, there are geological variables. For example, the mantle takes about 10,000 years to adjust to a given loading (WALCOTT, 1973). Thus, the bed elevations are time variables out of phase with the ice thickness (*e.g.* POLLARD, 1982). When the maximum depressions could be up to about 1,000 m this is a significant effect.

To the list of time varying geological conditions could be added bed deformability. A constant till layer is deformable or not depending on the basal temperature. Furthermore, it seems intuitively unlikely that all till layers are constant in area and thickness from one glaciation to the next or even within a given ice age. Possibly *some* deformable tills laid down by one episode of glaciation are utilized as deformable beds and removed by the next. This is liable to occur in BOULTON and JONES (1979) intermediate zones of net till deposition. For example, at present the glacial drift and/or marine sediment on parts of the Canadian Shield and in Hudson Bay is mostly thin compared to the sediment on the Prairies. Where it is thick in the Hudson Bay Lowlands it seems undisturbed (Shilts, personal communication). Thus, the hypothesized deformable Hudson Bay bed of this paper and the even more extensive area of deformable beds of BOULTON *et al.* (1985) must have resulted in removal of most of the responsible till depth.

More field and laboratory results are needed to estimate the thickness of the required deforming layers and to reconcile the deformation rates (and consequent till flux) with the supposed erosion rates. Much remains to be done in order to couple large ice sheets to their basal geology and to realistically include time into the models.

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APPENDIX

EFFECTS OF DEFORMABLE BEDS ON PLASTIC SOLUTION

The REEH (1982) theory is unchanged from equation A1 to A5.

The basic assumption of the theory is that flow lines may be defined as trajectories to the elevation contours of the ice-sheet surface, and that Equation (1) holds along such flow lines. Along a trajectory the surface gradient dE/ds is given by the equation

$$\left(\frac{dE}{ds}\right)^2 = \left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 \qquad \text{A1}$$

where x and y are orthogonal coordinates in a horizontal plane.

Introducing a quantity of dimension length $H_f = \tau_o/\rho g$ and substituting H for E-B, where B = B(x,y) is the elevation of the base of the ice sheet. Equations 1 and A1 may be combined to give the following differential equation for the elevations of a perfectly plastic three-dimentional ice sheet:

$$\left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 = \left(\frac{H_f}{E-B}\right)^2 \qquad \text{A2}$$

Equation (A2) may be solved by means of the method of characteristics. Applying the notation $p = \partial E/\partial x$ and $q = \partial E/\partial y$, equation A2 may be rewritten

$$p = \sqrt{(H_f/(E-B))^2 - q^2}, \qquad \text{A3}$$

assuming that the x-axis is oriented in such a way that $p \geqslant 0$.

According to KAMKE (1965, p. 66-67), the characteristic equations of the partial differential equation A3 are

$$\frac{dy}{dx} = \frac{q,}{P} \qquad \text{A3 A4}$$

$$\frac{dE}{dx} = \frac{p^2 + q^2}{p} = \frac{H^2_f}{(E-B)^2\, p}, \qquad \text{A5}$$

If τ_o or equivalently H_f is constant along a flow line then

$$\frac{dq}{dx} = \frac{(p^2 + q^2)\,(\partial B/\partial y - q)}{p(E - B)} = \frac{H^2_f\,(\partial B/\partial y - q)}{(E - B)^3 p} \qquad \text{A6}$$

If a flow line crosses one of the boundary lines in Figure 1 from deformable to a solid bed then equation A6 is not adequate. At this point the variability of τ_o or H_f must be introduced by assuming H_f is a function of x and y, *ie.* $H_f = H_f(x,y)$. Like REEH (1982) we want an expression for dq/dx.

As in the development of equation A6, one starts

$$\frac{dq}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y}\frac{dy}{dx}$$

$$\frac{\partial^2 E}{\partial x \partial y} = \frac{\partial^2 E}{\partial y \partial x} \quad \text{so that} \quad \frac{\partial q}{\partial x} = \frac{\partial p}{\partial y} \qquad \text{A7}$$

$$\text{and} \quad \frac{dp}{dx} = \frac{\partial p}{\partial y}dx + \frac{\partial q}{\partial y}dy$$

It is easy to find $\partial q/\partial y$ and $\partial p/\partial y$ of equation A7 from equation A3, and remembering $H_f = H_f(x,y)$ one quickly arrives at

$$\frac{dq}{dx} = \frac{H^2_f}{p(E-B)^3}\left(\frac{\partial B}{\partial y} - q\right) + \left(\frac{H_f}{p(E-B)^2}\right)\frac{\partial H_f}{\partial y} \qquad \text{A8}$$

Equation A8 is Reeh's equation A6 above with the additional term allowing for the change in H_f.

The problem of solving the non-linear partial differential Equation A2, is thereby reduced to solving simultaneously three first-order differential equations, of which Equation A4 defines the course of the flow-line projections on the xy-plane (From now on these projections will be simply referred to as flow lines.), and Equations A5 and A8 define the variations along the flow lines of the surface elevation and its gradient in the direction of the y-axis. It should be pointed out, that the flow lines are the only set of curves in the x - y plane along which the elevations of the ice sheet can be determined by integration.