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John T. Baldwin, "Model Theory and the Philosophy of Mathematical Practice: Formalization without Foundationalism."

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John T. Baldwin. *Model Theory and the Philosophy of Mathematical Practice: Formalization without Foundationalism*. Cambridge University Press 2018. 362 pp. $120.00 USD (Hardcover ISBN 9781107189218); $32.99 USD (Paperback ISBN 9781316638835).

This book deals with the scope and the role of contemporary—post-1970—model theory. It gives an intimate glimpse into the thinking of a practicing model theorist, and it will help philosophers of mathematics to situate modern model theory among branches of theoretical knowledge.

The term 'model theory' (or 'theory of models') was introduced by A. Tarski in 1954 to describe inquiry into formal properties of sets of sentences that imply properties of their models (302). Of course, properties of models of formal theories were investigated long before naming the area. The best-known results are the Skolem–Löwenheim theorems, the Gödel incompleteness theorems, and Robinson's non-standard analysis; all of them predate modern model theory by decades. Nonetheless, modern model theory thrives and it does so by limiting what it intends to investigate. It is obvious that model theory is concerned not merely with the model of the pure lower functional calculus, a question closed by Gödel's completeness theorem in 1931. But it is far from trivial that modern model theory focuses on *first-order theories*, moreover, on *tame theories*. 'Tameness' means that the theory lacks an apparent capability to create a Gödel sentence; perhaps, it cannot define a pairing function, hence, supports a notion of dimension in the geometrical sense. This limitation excludes from consideration some of the primary worries in the philosophy of mathematics. Baldwin explicitly tries to avoid any questions pertaining to the foundations of mathematics. Of course, the 'Gödel phenomenon' (i.e., the syntactic incompleteness of a mathematical theory) appears not only in first-order arithmetic—whether Peano's or Robinson's—but also in first-order set theory, in particular, within ZFC (Zermelo–Fraenkel set theory with the axiom of choice). One might feel some unease at this point, because although the first-order theories themselves are typically countable sets, the discussion of their models assumes a grasp of ZFC (and possibly, some of its extensions). But Baldwin assuages these fears with the discussion of Skolem's views on the relativity of set-theoretic notions (90-1). Modern model theory does not rely on ZFC (or its extensions) as a formalized theory; rather, ZFC is an informal background theory. (This may also explain why Wolfram's MathWorld [https://mathworld.wolfram.com] situates 'Model theory' under *Set theory*, which in turn is under the *Foundations of Mathematics*.)

The choice of first-order theories to formalize areas of mathematics could be viewed as a strength with respect to constructing verifiable proofs. However, modern model theory is not concerned with either proofs or with the consistency of theories. (Indeed, the Reinhardt cardinal is mentioned (60) as a source of discovery, not as a troublesome incident.) First-order theories are exceedingly rarely categorical, which may be the reason why model theorists shy away from trying to pinpoint the mathematical structure that a theory is talking about. Instead, Baldwin assumes that practicing model theorists have a handful of canonical structures that are directly accessible to them. These are: natural numbers, integers, rationals, reals, and complex numbers together with Euclidean geometry. (In the 21st century, p-adic numbers and the complex field with exponentiation are added to the list.) These structures are given through intuition, which Baldwin seems to equate with 'visualization' not only within Euclidean geometry, but in set theory too (79). Of course, *six* or *eight* canonical structures are not going to take us far, especially, because the vocabulary that is used to formulate theories about the structure is crucial (44). The success of a formalization of an area of mathematics hinges on the selection of a suitable vocabulary. For example, the 'wildness' of Peano arithmetic is mitigated in Pressburger's arithmetic, which excludes multiplication from the language (and is a decidable theory). However, the problem of the lack of categoricity persists, and it is more pressing for a model theorist than the presence or absence of a syntactic property (e.g., decidability).

Baldwin acknowledges that leading model theorists (e.g., Morley and Keisler) considered model theory of first-order theories to be a finished subject in the late 1960s (178), and were expecting further development in the area of infinitary logics, mainly, in $\boldsymbol{L}_{\omega}1_{,\omega}$. However, 'At present, there is essentially no model theory of second order logic, a richly developed model theory of first order logic, and a comparatively rudimentary model theory for infinitary logic.' (19) The aforementioned development resulted from further efforts along the 'old model theoretic lines,' which tries to find useful properties of theories, where 'usefulness' means interesting implications for the models of the theory. Keisler introduced the notion of what became known as *Keisler order* while Shelah formulated his stability program leading to classification theory. Shelah wanted to find *dividing lines* for the class of countable first-order theories that would allow all the theories to be put under one of two headings 'classifiable' and 'creative.' The latter kind of theories are badly behaved, because they have a great variety of models, which cannot be systematized. Classifiable theories, on the other hand, admit a structure theory. (Shelah's famous 'Main Gap Theorem' can be thought of as saying that a complete countable theory with further properties (shallow, superstable, etc.) guarantees that there is a structure theory. (298)) Routine techniques in abstract algebra allow one to take (possibly small) structures, such as the 2-element Boolean algebra, and construct larger structures (e.g., as their direct product). Baldwin explains that the aim of stability theory is not to find small 'atomic' structures (like the only sub-directly irreducible Boolean algebra **2**) that serve as building blocks of larger structures, but to *decompose* large models into a tree of elementarily equivalent models, so that the larger models turn out to be direct limits of the trees.

Categoricity is not attainable for first order theories – except in rare and uninteresting cases. But it is not something a modern model theorist is reaching for. Baldwin argues that 'categoricity in power' is more important (49). (The latter is a shorthand for a theory being κ-categorical for every infinite cardinal κ.) If categoricity in power does not obtain, then the model theorist would like to count the number of κ-sized models (up to isomorphism). In addition, if that attempt fails too, then he would like to be able to relate the models to each other. The discussion of second-order categoricity and of internal categoricity reveals that model theorists are not concerned about describing a unique structure such as the natural numbers or the realm of sets. Clearly, Baldwin is not interested in some of the traditional problems that philosophers of mathematics ponder about such as the nature of mathematical objects. Yet the frequent appeals to intuition and visualization as ways to interact with mathematical structures leaves one wondering if mathematics, after all, is an exercise in subjectivity, especially when the foundational problems are readily brushed aside.

The third part of the book is devoted to geometry. Of course, Euclidean geometry is a canonical structure, but the connections to the title of the book go deeper than that. Geometry is the prototypical mathematical theory without the 'Gödel phenomenon' that can be proved consistent by finitary methods. Some milestones in geometry are Euclid's *Elements* (which served as the paragon for rigorous presentation of any body of knowledge for centuries), Descartes's coordinate system, Hilbert's re-axiomatization of Euclidean geometry, Tarski's axiomatization of Euclidean geometry, and George Birkhoff's axiomatization of Euclidean geometry. A discussion of these geometries in terms of the underlying 'data set' and axioms allows Baldwin to highlight ideas such as the (im)modesty of an axiomatization and the (im)purity of proofs. The chapters on geometry are teeming with historical details, and even with quotes from definitely non-contemporary authors (like Leibniz, Sylvester, and Dedekind). Obviously, Euclidean geometry has been around much longer than stability theory, and many more philosophers have written about it. This provides Baldwin with ample opportunity to engage with the views of philosophers.

The book, or perhaps modern model theory itself, has some cons such as repeat uses of the same term with unrelated technical meanings. ('Tame,' 'geometry' and 'type' are the most salient examples here.) The very limited breadth of modern model theory is also surprising. Second-order logic and infinitary logic are at least mentioned by Baldwin, but there is no consideration of logics between first- and second-order, or of models of theories formulated in logics that are not the 2-valued (so-called 'classical') logic. For instance, intuitionistic logic grew out of a particular view on mathematical practice.

Baldwin adopts an informal style in this book, with fluid transitions between areas of mathematics. Reading the book does not require thorough familiarity with modern model theory or mathematics, because there is only a minimal amount of notation used, and some notions are defined in footnotes. (The bibliography—comprising well over 500 items—is a useful source for further reading.) A reader interested in the history of mathematics will find enjoyable sketches of the development of certain concepts (e.g., the coordinatization in geometry). With Baldwin's help, we get a peek into modern model theory, which provides many potentially exciting problems for philosophical research, from Shelah's monster domain to the idea of formalism-freeness.

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