Objective Reconstructions of the Late Wisconsinan Laurentide Ice Sheet and the Significance of Deformable Beds

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Résumé de l'article
À partir d’un modèle théorique tridimensionnel de plasticité de la glace, de la topographie actuelle (sur un canevas de 50 km2), du nouveau consensus quant à la limite maximale de la marge glaciaire (PREST, 1984) et d’une carte des seuils de plasticité de la glace, les auteurs ont élaboré des modèles de la calotte glaciaire laurentidienne. On a donc reconstitué par ordinateur une calotte asymétrique à domes multiples, sans idée préconçue quant aux directions de l’écoulement des glaces. On a évalué les conséquences de la présence éventuelle de lits non résistants en se fondant sur les très bas seuils de plasticité de la glace proposés par MATHEWS (1974). En raison des bas seuils de plasticité (lits non résistants), les modèles démontrent qu’une glace peu épaisse couvrait les Prairies et la région des Grands Lacs, ainsi que la baie d’Hudson, dans un des deux cas. La prise en considération de régions à bas seuils de plasticité (lits non résistants) montre également la présence de pentes faibles et des changements brusques de direction de l’écoulement glaciaire. Dans certains cas, de grands courants glaciaires se manifestent le long des limites entre les endroits où les seuils de plasticité sont normaux (lits rigides) et les endroits où les seuils de plasticité sont bas (lits non résistants). Les modèles obtenus par ordinateur sont ensuite comparés aux reconstitutions de SHILTS (1980) et de DYKE et al. (1982), élaborées à partir des données géologiques.
OBJECTIVE RECONSTRUCTIONS
OF THE LATE WISCONSINIAN
LAURENTIDE ICE SHEET
AND THE SIGNIFICANCE
OF DEFORMABLE BEDS

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ABSTRACT A three dimensional steady state plastic ice model; the present surface topography (on a 50 km grid); a recent consensus of the Late Wisconsinan maximum margin (PREST, 1984); and a simple map of ice yield stress are used to model the Laurentide Ice Sheet. A multi-domed, asymmetric reconstruction is computed without prior assumptions about flow lines. The effects of possible deforming beds are modelled by using the very low yield stress values suggested by MATHEWS (1974). Because of low yield stress (deforming beds) the model generates thin ice on the Prairies, Great Lakes area and, in one case, over Hudson Bay. Introduction of low yield stress (deformable) regions also produces low surface slopes and abrupt ice flow direction changes. In certain circumstances large ice streams are generated along the boundaries between normal yield stress (non-deformable beds) and low yield stress ice (deformable beds). Computer models are discussed in reference to the geologically-based reconstructions of SHILTS (1980) and DYKE et al. (1982).
INTRODUCTION

Why another ice sheet reconstruction

Andrews recently reviewed Late Wisconsinan ice sheet reconstructions (Andrews, 1982) and noted that many theoretically-based reconstructions tend to generate symmetrical ice cover with thick ice cover over the Canadian prairies and Hudson Bay (PateraSon, 1972; SuggDen 1977; Denton and Hughes, 1981; Boulton et al., 1985; Hughes 1985). In contrast Peltier's (1981) reconstruction based on uplift data suggests generally thinner ice and very thin ice cover over the prairies. Recently REEH et al. (1983) and Boulton et al. (1985) have introduced deformable beds into the ice flow modelling exercise. These beds have the effect of producing areas of thinner ice and removing much of the symmetry in the reconstructions. The Denton and Hughes model can also introduce deformable beds but they have not yet done much with the concept. Andrews also notes that some models rest sensitively on unknown input parameters such as accumulation patterns at 18 ka BP and, yet others, by using some of the geological evidence to infer major flow lines, leave themselves with no independent evidence to check the results of the model.

We present here an expansion of the (REEH et al., 1983) computer reconstructions of the Late Wisconsinan ice cover. We use a very simple ideal plastic ice rheology that is rather insensitive to unknown parameters and takes as input only the margins of the ice sheet, the present day topography, and an assumed yield shear stress, \( \tau_o \). A flow line calculation begins at a point on the margin and integration of the equation proceeds up slope using only the margin's shape, the underlying topography and the yield stress. The surface elevations and trajectories are calculated for many flow lines around the entire margin. No assumptions are made in advance about the margins or the bed topographies. The original development by REEH (1982) developed and used an earlier version of the program on the Greenland ice cap, successfully reproducing all the major ice divides, ice streams and centres. His calculated flow line trajectories were accurate and the model's surface elevations were at most in error by 10%.

Also we attempt to allow for the effects of deformable beds in the western plains, Hudson Bay and the Great Lakes regions of the ice sheets. Deformable beds are modelled here by simply using small values of yield stress, \( \tau_o \), for the ice. The computed ice sheets are non-symmetric with thin ice over the prairies, thick ice on the Canadian Shield, and in the case of low \( \tau_o \), Hudson Bay beds with thin ice in Hudson Bay. The predicted domes, ice divides and ice streams can be favourably compared to much of the surface glacial morphology.

We think that the model provides a robust, objective, and economical tool for linking and cross-checking the two main types of geophysical field evidence, namely ice margins and flow direction indicators (i.e. striae, erratics trains, etc.).

THE MODEL

The model is detailed by REEH (1982) and the following summarizes it. Ice is taken as an ideal plastic with a yield stress \( \tau_o \) normally in the range 0.5 to 1.5 bars (1 bar = 10^5 N m^-2). Neglecting longitudinal stress gradients and transverse shear stress, the ice elevation \( E \) as function of distance \( S \) measured positive up a flowline is governed by:

\[
\frac{dE}{dS} = \frac{H_s}{H}
\]

with \( H_s = \tau_o/\rho g \) (\( H_s \) has a dimension of length.)

\[
\rho = \frac{2\tau_o L}{gLp}
\]

with \( H_s \), the maximum thickness at the divide

\[
L = \text{distance between the divide and the margin.}
\]

If the accumulation rate, \( a \) (m ice per year) and basal temperature \( T \) (°C) are constant, the equivalent plastic yield stress can be estimated by (REEH, 1982).

\[
\tau_o = \left( \frac{(n + 2)gap}{4B_0 \exp(kT)} \right)^{1/(n + 1)}
\]

The quantities \( n, B_0 \), and \( k \) come from Glen's Flow Law for ice and are 3, 0.5 bar^{-1} year^{-1}, and 0.2 deg^{-1} respectively (REEH, 1982).

Equation 3 is not used to calculate the \( \tau_o \)'s but serves as a guide in assessing how to vary \( \tau_o \) when accumulation and temperature \( T \) change. \( \tau_o \) is seen to be rather insensitive to \( a \) and \( T \). In the program \( \tau_o \) is an average value for an entire flow line, unless it crosses the heavy hatched lines of Figure 1.

The computational algorithm comes from the assumption that equation 1 is true along any flow line and further that

\[
\nabla E \times \overrightarrow{ds} = 0
\]

Equation 4 states that vector increments \( \overrightarrow{ds} \) along a flow line are perpendicular to the surface slope. Equations 1 and 4 can be solved by the method of characteristics for arbitrary margins and bed topographies. The original development by REEH (1982) must be extended slightly here (see Appendix) to include terms that become large when the yield stress \( \tau_o \) changes significantly along a flow line, e.g. when a flow line crosses from (deformable to non-deformable beds) normal to low \( \tau_o \) ice.

The solution allows the bed to be depressed isostatically to \( B_0 \) (relative to present sea level) using

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\[
L = \text{distance between the divide and the margin.}
\]
where \( B_a \) is the present "unloaded" elevation

\[ B_a = B_a - H \frac{\rho}{\rho_r}, \]

where \( B_a \) is the present "unloaded" elevation
\( \rho_r \) is the density of the upper mantle (3300 kg m\(^{-3}\)).

Since time does not enter what is basically a force balance solution, the isostatic adjustment has no time delay.

**THE MARGIN MAP**

The time instant we attempt to reconstruct is nominally the 18 ka extent of the Laurentide Ice Sheet. A great deal of uncertainty still exists about the northern and northeastern positions of this margin (ANDREWS, 1982). Probably this maximum margin was not synchronous. The age spread around 18 ka for maximum margins seems to be ± 4 ka (Prest, personal communication).

We chose a slightly smoothed version of one of two proposed alternatives for the Late Wisconsinan. PREST (1984) refers to them as the minimum and maximum concepts. Even if the entire margin is not a synchronous feature the reconstruction is valid for local areas scaled by lengths of margin that are known to be synchronous. We present the minimum concept results here. The maximum concept results with deformable Prairie and Great Lakes beds have been presented in a poster session (REEH et al., 1983) (see cover).

**BED TOPOGRAPHY MAP**

A one-square degree data base (USDMAC, 1976) is used to generate a regular 50 by 50 km grid of the crusts' surface elevations. The grid is with respect to a cartesian coordinate system laid over a Lambert conformal conic projection with two standard parallels at latitudes 49° and 77°. Linear distortions are less than 3%. Where the ice margins are beyond the present shore one should unload the undersea topography before depressing it, but since the depths involved are small we neglect this effect. All interpolations of surface topography are done using spline functions. No attempt is made to allow for any residual rebound left in the present topography. This is largely because there is no agreement on the magnitude or response time of the long-term mantle readjustment and, even if there was, we would not know the phase of the readjustment at 18 ka BP because that would require a loading history stretching back over 100 ka.

**THE YIELD STRESS MAP**

Ideally the equivalent plastic yield stress map would be represented with the same detail as the topography and vary with basal temperature accumulation and bed type. Since there is little hard data we chose a very simple map of \( \tau_0 \) (Fig. 1).

REEH (1982) used the model to generate accurate surface elevations, ice divides and ice streams of present-day Greenland. He obtained best results for flow lines in north and central Greenland with \( \tau_0 = 0.65 \) and 0.9 bars respectively. These flow lines include along their lengths variable accumulation and basal temperatures, but the insensitivity of \( \tau_0 \) (see equation 3) allowed the use of a single average value for each region. The northern flow lines have a lower \( \tau_0 \) (in line with equation 3) largely because north Greenland has low accumulation, i.e. 0.15 m yr\(^{-1}\) (ice equivalent) as opposed to 0.3 for central Greenland.

Our \( \tau_0 \) assignment for coastal high accumulation regions and northern or inland moisture starved regions follows the Greenland calibration (REEH, 1982). The ice thicknesses depend on the choice of \( \tau_0 \), but experiment shows that the flow directions are insensitive to \( \tau_0 \) in the normal range.

Over large areas in western and south central North America we take \( \tau_0 \) very much smaller than ‘normal’ because of the thin ice profiles suggested by field observations (MATHEWS, 1974) and theory (BOULTON and JONES, 1979). Small \( \tau_0 \) does not imply that the ice itself is abnormally soft, but more likely that the bed sediments are themselves deformable. We allow for this by using small \( \tau_0 \)’s derived from Mathews’ inferred surface profiles. Much of the boundary between low \( \tau_0 \) and normal \( \tau_0 \) is taken as the edge of the Canadian Shield and is shown in Figure 1a by a hatched line. The “normal” \( \tau_0 \) values (0.5 bars < \( \tau_0 \) < 1.5 bars) are used...
over granite and plutonic rocks and the low $\tau_0$ values ($\tau_0 < 0.2$ bars) are confined to Proterozoic rocks. Proterozoic sediments and unconsolidated sediments. Two complete reconstructions were done, one with ‘normal’ yield stresses for the Hudson Bay — Hudson Strait area and one with $\tau_0 = 0.14$ bars. MATHEWS (1974) analysis of ice age nunataks in the Prairies allowed empirical estimation of $\tau_0$ on the Prairies and BOULTON and JONES (1979) give some justification for the low $\tau_0$ values in the Great Lakes region. Thus, Figure 1a has some empirical validity.

There is no similar direct evidence that the Paleozoic and Proterozoic beds under and around Hudson Bay were deformable. Although these sedimentary rocks are rather weak and relatively easy to break up (Shilts, personal communication), it is hard to imagine that they are deformable in the same sense as the thick, loose drift of the North American Plains. Also the Hudson Bay Lowland glacial sediment cover, though thick, does not show signs of deep disturbance (Shilts, personal communication) as would be expected with the prairie type of deforming beds. With these reservations in mind we try the experiment of assigning relatively low values of $\tau_0$ (1.4 bars) to Hudson Bay and the Lowlands. BOULTON et al. (1985) feel the same way but do not go into details as to how or why they obtain the equivalent of our low $\tau_0$. Figure 1a shows the $\tau_0$ map used with “normal” $\tau_0$ values for Hudson Bay and Figure 1b with small values for the Hudson Bay region. Given that water pressure is a key factor in the deformation of beds (BOULTON and JONES, 1979), the ice's basal temperature must be at the pressure melting point. If such beds are frozen they will not easily deform and the appropriate $\tau_0$ would then be in the normal range. Thus, one could hypothesize Figure 1a as a frozen Hudson bed case and Figure 1b as a melted Hudson bed case.

Basal temperatures under large ice masses are dependent on many factors (PATERSON and CLARKE, 1978) such as surface air temperature, surface melt, thickness, accumulation or ablation rate, ice velocity, ice deformation heating, and geothermal heat flux. For basal temperatures of thick ice there is a long-time delay (a few thousand years) between changes in surface variables and changes at the bottom. Later we will come back to the possibility that the Hudson Bay area's basal temperature went through episodes at and below the melting point with the beds consequently alternating between deformable and hard states. Finally, we assume that ice rheology was like present Holocene ice though there is some evidence that Wisconsinan ice laden with high microparticle concentrations is “softer” (PATERSON, 1977, 1981; FISHER and KOERNER, submitted; DAHL-JENSEN, 1985).

THE RECONSTRUCTIONS

Figure 2 presents the reconstruction using a hard or frozen Hudson Bay and Figure 3 using a low $\tau_0$, soft Hudson Bay. Both assume very low yield stresses for the Prairies and Great Lakes regions. In both versions there are several identifiable ice centres and ridges. There is no trace of a single large ice centre, but rather multiple centres and ridges. The ice divides are heavy lines, elevations at 400 m intervals are medium solid lines, and the boundary between deformable and hard beds is given by dotted lines. Figure 2 shows the ice sheet consisting of three connected centres with a number of lateral spur ridges. Figure 3 has four major centres arranged in a U pattern around Hudson Bay. The maximum concept reconstruction (see cover), which assumed a hard Hudson Bay bed, had the same centres and major lateral ridges as the minimum concept reconstruction of Figure 2. The Laurentide ice volumes calculated for the minimum and maximum reconstructions are $21.1 \times 10^6 \text{ km}^3$ and $25.9 \times 10^6 \text{ km}^3$ respectively, assuming a hard Hudson bed.

The low $\tau_0$ Hudson Bay reconstruction has all the same centres, etc. as the frozen bed version, only the centres are farther away from the Bay and about 400 m lower. The Bay itself is an “ice sink” instead of being a high ridge area as in the normal $\tau_0$ reconstruction. The hard bottomed Hudson ridge K-H of Figure 2 seems to “become” the two widely-separated ridges K-H and Q-U of Figure 3. The total ice volume calculated for the minimum concept low $\tau_0$ Hudson Bay bed reconstruction is $18.0 \times 10^6 \text{ km}^3$. For comparison, two recently proposed reconstructions, based on geological evidence, are presented in Figures 4 and 5. Figure 4 is due to DYKE et al. (1982) and is in good agreement with our deformable Hudson Bed reconstruction (Fig. 3). Figures 5, from SHILTS (1980), differs from Dyke et al. and our reconstructions in one major respect. Using the fact that some erratics found on the southwest side of Hudson Bay and well into Ontario and Manitoba originated on the east side of Hudson Bay, Shilts invokes stable long east-to-west flow lines across southern Hudson Bay during much of the Late Wisconsinan. Dyke et al., on the other hand, explain the erratics by a re-entrainment hypothesis that moves these rocks westward across the Bay over several cycles of growth and decay of the ice sheet. The Hudson Dome of Figure 4 precludes such long flow lines in the Dyke et al. reconstruction. DYKE et al. (1982) hypothesis in principle is attractive to us but one of their main arguments against Shilts’s long flow lines is (in light of possible deformable beds) not as strong as it was. DYKE et al. (1982) considered a 1600 km Shilt’s flow line from the Labrador ice divide to Lake Winnipeg and used equation 2 (with “normal” values of $\tau_0$) to calculate that the elevation at the Labrador divide (Fig. 5) was 6 km. They calculated the elevation of the divide to be only about 3 km using the eastward flow lines from the divide 500 km to the coast of Labrador. This lack of symmetry about the Labrador ice divide and the resulting discrepancy in calculated divide elevations is a telling argument against the long Shilt’s flow lines if $\tau_0$ is “normal” for both east and west flowing ice. But, if the west flowing ice is mostly over deformable beds with $\tau_0 = .14$ bars, and the east flowing ice mostly over hard beds $\tau_0 = .54$ to .81 bars, then one would expect there would be an asymmetry in the length of the flow lines and the ratio of the lengths of west flowing to east flowing lines would be the inverse of the ratios of the effective yield stresses, i.e. 3.9 to 5.8. The actual ratio for the lengths of Shilt’s flow lines (west/ east) is about 3.2. Also, with deformable beds included, the inconsistency in the calculated elevation of the Labrador ice divide is resolved. However, while our model with a soft Hudson
bed certainly generates a feature like the Labrador divide (Q-U in Fig. 3), it does not produce the 1600 km long westward flow lines to Lake Winnipeg. Possibly there was a major bridge of low yield stress bed material between Hudson Bay and the Prairies that we should have included or maybe the erratic movements and these long apparent flow lines are not the result of a long-lived steady state ice sheet configuration. This will be discussed later.

DISCUSSION

WHAT HAPPENS AT $\tau_0$ BOUNDARIES BETWEEN HARD AND SOFT BEDS?

In the Appendix we describe what happens in this model when a flow line crosses from a hard bed (modelled by $\tau_0 \sim 1$) to a deformable bed (modelled by $\tau_0 << 1$) region. The ice surface slope changes by a factor $(\tau_{\text{soft}}/\tau_{\text{hard}}) \sim 1/2$ to $1/10$.
and the direction of the flow lines can change dramatically. In low $\tau_0$ areas ice flow is very sensitive to topography and is more readily diverted to low areas or around uplands whereas, in hard bottom areas, the bottom topography is only partially controlling flow direction.

The deformable Prairie beds thus produce the relatively thin ice cover and the remarkable abrupt direction changes (Fig. 6) along the boundary between the soft Prairie sediments and hard Canadian Shield rocks. These model-generated results would seem to be in excellent agreement with flow directions on the western part of DYKE et al. (1982) map (Fig. 4) even to the correct placement of the Prairie ice divide labelled P in Figures 2, 3 and 4.

Given certain geometric relationships between the flow line directions, topography and the $\tau_0$ boundary, we found areas where there was a strong convergence or focussing of the flow lines. Figure 6 shows such a very large ice stream generated by the model. One can speculate that the large lakes (e.g. Lake Winnipeg) are caused by this ice stream. Similar remarkable ice streams are model-generated flowing north on the east side of a soft-bottomed Hudson Bay and on the south side of Foxe Dome flowing east (Fig. 3).

SPECULATIONS ABOUT THE RECONSTRUCTIONS AND THE LABRADOR ERRATICS

What follows is a highly speculative effort to reconcile our steady state reconstructions with Shilt's erratics. If the bed under Hudson Bay is frozen solid, and is thus not deformable, then the steady state model ice sheet would look like Figure 2, whereas a long enough interval of time with a melted-

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**FIGURE 3.** Computer reconstruction of the Laurentide Ice Sheet at 18 ka with deformable Hudson Bay, Prairie and Great Lakes beds. Surface elevations are in 100's of metres above present sea level. Lettered features are as in the caption of Figure 2. Ice flow lines are perpendicular to the elevation contours. Heavy lines denote ice divides and dotted lines mark the boundary between hard and deformable beds.
deformable Hudson Bay bed would produce a model steady state shown in Figure 3. One might consider these two possible states as extremes between which the real time varying ice sheet oscillated. If a case can be made for this hypothesis then one could speculate that the Q-U Labrador ice divide of Figure 3 migrates from its asymmetric soft Hudson Bay position to the more symmetric central K-H position of Figure 2 which assumes a frozen or hard Hudson Bay bed. The migration of the Labrador ridge would sweep all the southern Hudson ice and its contents westward, and the position of the K-H ridge of Figure 2 would result in the continued westward movement of debris originating on the east side of the Bay.

There are some lines of evidence that suggest several cycles of growth and decay of the Hudson Bay ice during the Wisconsinan ice age. ANDREWS et al. (1983) maintain from amino acid analysis of shells that southern Hudson Bay was ice-free at 35 ka, 75 ka, and 105 ka; i.e. that there were three complete cycles of growth and decay of ice over Hudson Bay during the last glaciation. There are, however, some as-
sumptions in the dating procedure, in particular one must know the whole temperature history of the samples to interpret the amino acid ratios.

BUDD and SMITH's (1981) "climate-ice sheet" model of the Wisconsinan glaciation could vary with time, and the main external forcing function, the total summer radiation over the last 200 ka years, generated three cycles of growth and decay. ANDREWS and MAHAFFY (1976) modelled the initial growth of the Wisconsinan ice sheet and found the Labrador centre grew from nothing westward right across Hudson Bay in only 7000 years. Certainly the Hudson Bay ice and, in fact, the entire ice sheet can disappear quickly (within a few thousand years), so there is time enough for several cycles during the last ice age.

The realistic calculation of basal temperatures of large ice sheets is beyond the scope of this paper, but since basal temperatures at the pressure melting point are needed to activate potentially deformable beds the topic is important. At this time not enough is known about the mechanical and thermal regimes of ice moving over active deformable beds to quantitatively evaluate the stability of the soft Hudson bed mode (Fig. 3). There is some guidance though for the other case of a frozen non-deformable Hudson bed (Fig. 2).

Near the central Hudson Bay ridge with the following plausible range of conditions, surface elevation 2800 m, thickness 4200 m ice surface air temperature -35°C, accumulation 0.20 to 1.0 m a⁻¹, yield stress τ₀ > 0.7 bars and geothermal heat flux 4.18 x 10⁻² W m⁻² and assuming a steady state condition with no horizontal advection terms, one would calculate using the simple heat transfer theory (PATERSON, 1981) that the basal ice was frozen and thus the bed non-deformable. However, CLARKE et al. (1977) have demonstrated that because of strain heating the solution of the steady state is multi-valued, and thus unstable under certain conditions. For example, under the above conditions, the solution is unstable and an initially-frozen bottom would warm up and melt within a few thousand years. One can conclude that there is a strong possibility that basal temperature under the hard Hudson Bay reconstruction would always tend to the melting point and thus set the stage for possible bed deformation and "collapse" to the soft bed case.

**OTHER WORK AND FUTURE WORK**

Recently BOULTON et al. (1985) have modelled the Laurentide Ice Sheet with and without deformable beds, and their latter reconstruction seems to accommodate the long flow lines of Shilts into a steady state model. The details of their reconstruction are a bit sketchy and their ice flow lines do not exhibit the direction changes one expects at the boundaries between hard and deformable beds. This is particularly apparent in the Prairie region of North America where their flow line directions differ from those inferred from the surface geology (DYKE et al. 1982) (Fig. 4).

In trying to reconcile all the glacial landforms with steady state models, we and others may be asking too much of the steady state. Even a sequence of steady states is no substitute for an explicit time variable. In addition to the climatic time variables of air temperature, accumulation, melt, equilibrium line elevation, and sea level, there are geological variables. For example, the mantle takes about 10,000 years to adjust to a given loading (WALCOTT, 1973). Thus, the bed elevations are time variables out of phase with the ice thickness (e.g. POLLARD, 1982). When the maximum depressions could be up to about 1,000 m this is a significant effect.

To the list of time varying geological conditions could be added bed deformability. A constant till layer is deformable...
or not depending on the basal temperature. Furthermore, it seems intuitively unlikely that all till layers are constant in area and thickness from one glaciation to the next or even within a given ice age. Possibly some deformable tills laid down by one episode of glaciation are utilized as deformable beds and removed by the next. This is liable to occur in BOULTON and JONES (1979) intermediate zones of net till deposition. For example, at present the glacial drift and/or marine sediment on parts of the Canadian Shield and in Hudson Bay is mostly thin compared to the sediment on the Prairies. Where it is thick in the Hudson Bay Lowlands it seems undisturbed (Shilts, personal communication). Thus, the hypothesized deformable Hudson Bay bed of this paper and the even more extensive area of deformable beds of BOULTON et al. (1985) must have resulted in removal of most of the responsible till depth.

More field and laboratory results are needed to estimate the thickness of the required deforming layers and to reconcile the deformation rates (and consequent till flux) with the supposed erosion rates. Much remains to be done in order to couple large ice sheets to their basal geology and to realistically include time into the models.

ACKNOWLEDGEMENTS

Frequent discussions and critical reviews by Stan Paterson of Paterson Geophysics Inc. and by Lynda Dredge, Arthur Dyke, Robert Fulton, Vic Prest, William Shilts and Jean-Serge Vincent of the Geological Survey of Canada were essential for development of this model by the non-geologist authors. Also financial help from the Polar Continental Shelf Project, Energy, Mines and Resources Canada allowed one of us (N. Reeh) to visit Canada for a month during which much of the work was done. We also thank André Maisonneuve of the Surveys and Mapping Branch, EMR, for the shaded relief drawing that appears on the cover.

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APPENDIX

EFFECTS OF DEFORMABLE BEDS
ON PLASTIC SOLUTION

The REEH (1982) theory is unchanged from equation A1 to A5.

The basic assumption of the theory is that flow lines may be defined as trajectories to the elevation contours of the ice-sheet surface, and that Equation (1) holds along such flow lines. Along a trajectory the surface gradient dE/ds is given by the equation

\[
\frac{dE}{ds} = \left( \frac{\partial E}{\partial x} \right)^2 + \left( \frac{\partial E}{\partial y} \right)^2
\]  

where \( x \) and \( y \) are orthogonal coordinates in a horizontal plane.

Introducing a quantity of dimension length \( H_i = \tau_0/\rho g \) and substituting \( H \) for \( E-B \), where \( B = B(x,y) \) is the elevation of the base of the ice sheet. Equations 1 and A1 may be combined to give the following differential equation for the elevations of a perfectly plastic three-dimensional ice sheet:

\[
\left( \frac{\partial E}{\partial x} \right)^2 + \left( \frac{\partial E}{\partial y} \right)^2 = \left( \frac{H_i}{E-B} \right)^2
\]

Equation (A2) may be solved by means of the method of characteristics. Applying the notation \( p = \xi E/\xi x \) and \( q = \xi E/\xi y \), equation A2 may be rewritten

\[
p = \sqrt{(H_i/(E-B))^2 - q^2}
\]

assuming that the x-axis is oriented in such a way that \( p \geq 0 \).

According to KAMKE (1965, p. 66-67), the characteristic equations of the partial differential equation A3 are

\[
\frac{dy}{dx} = \frac{q}{p}, \quad \frac{dx}{dy} = \frac{P}{q}
\]

\[
\frac{dE}{dx} = \frac{p^2 + q^2}{p} \left( \frac{H_i}{(E-B)^2} \right)
\]

If \( \tau_0 \) or equivalently \( H_i \) is constant along a flow line then

\[
\frac{dq}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} \frac{dy}{dx}
\]

\[
\frac{d^2 E}{dx dy} - \frac{d^2 E}{dy dx} \quad \text{so that} \quad \frac{\partial q}{\partial x} - \frac{\partial p}{\partial y} = -\frac{\partial p}{\partial y} dx + \frac{\partial q}{\partial y} dy
\]

\[
\frac{dp}{dx} = \frac{\partial p}{\partial y} dx + \frac{\partial q}{\partial y} dy
\]

Equation A8 is Reeh's equation A6 above with the additional term allowing for the change in \( H_i \).

The problem of solving the non-linear partial differential Equation A2, is thereby reduced to solving simultaneously three first-order differential equations, of which Equation A4 defines the course of the flow-line projections on the xy-plane (From now on these projections will be simply referred to as flow lines.), and Equations A5 and A8 define the variations along the flow lines of the surface elevation and its gradient in the direction of the y-axis. It should be pointed out, that the flow lines are the only set of curves in the x - y plane along which the elevations of the ice sheet can be determined by integration.