Laval théologique et philosophique

Abstraction from Matter (II)

Charles De Koninck

Volume 16, numéro 1, 1960

URI : https://id.erudit.org/iderudit/1019986ar
DOI : https://doi.org/10.7202/1019986ar

Aller au sommaire du numéro

Éditeur(s)

Laval théologique et philosophique, Université Laval

ISSN

0023-9054 (imprimé)
1703-8804 (numérique)

Découvrir la revue

Citer cet article

De Koninck, C. (1960). Abstraction from Matter (II). *Laval théologique et philosophique*, *16*(1), 53–69. https://doi.org/10.7202/1019986ar

Abstraction from Matter*

XIV. SOME MEANINGS OF THE WORD ' ABSTRACTION '

Having examined the word ' matter ' in ' abstraction from matter ' we must now turn to ' abstraction. ' Like the Greek term *aphairesis*, the Latin *abstractio*, from *ab* and *trahere*, meant the process of drawing one thing away from another, as to pull an apple from a tree, or hew stone from stone.

1. ' *Abstraction* ' *is an analogous term*

By extension, this term is applied to knowledge. I can taste an apple without seeing it, or see it without tasting. In reality the apple has both colour and flavour. To perceive the one without the other is to abstract. The same term acquires a new meaning again when applied to understanding. For of objects which in reality are together, one may be considered separately, so long as the understanding of one part or aspect of the thing is not essential to an understanding of the other. For instance, Socrates is stout, the husband of Xanthippe, and a player of the flute. In his particular case all these things go together, so that it would be false to say that he is stout but not a husband, stout and husband but not a player of the flute. Yet the mind can consider each of these attributes of Socrates, one apart from the other, for a man can possess one of them without the other. In other words, our mind can be brought to bear upon one of them, abstracting from the other, even though the latter be in fact conjoined to the former. If this were not so it would be false to say that Socrates is a husband without saying that he is stout, or that he lives in Athens, and so forth.

Now there is still another way the mind performs abstractions, namely, when something can be considered apart from something else because the one is prior to the other, eventhough in subject they be one and the same thing. For instance, I can consider man as an animal, abstracting from the fact that he is an animal of a very special kind ; and I can consider man without considering *this* one who is Socrates. But I cannot conceive man without conceiving animal, nor this man without conceiving man. Animal is prior to man inasmuch as an animal is not necessarily a man, even as a man is not necessarily Socrates. Both examples convey abstraction of universal from particular. In the first case we abstract a universal, animal, from a less universal, man ; in the second, the particular is a singular. It is likewise called abstraction of the whole from the subjects or ' subjective

---

* See the first part of this study in *Laval théologique et philosophique*, Vol. XIII, 1957, n° 2, pp.133-196.

parts ' of which it can be said. (This term ' part ' is an analogical term, for Socrates is not part of man in the sense that his head is part of the whole that is Socrates ; nor is horse a part of animal in this early sense of part.)

The term ' abstraction,' then, is plainly an analogical one. It has a further meaning still in the special case of mathematics. Just as we can consider animal without man, and man without Socrates, we can also consider quantity without the qualities that attend it. This is seen in the fact that we grasp and define numbers, point, line, surface and volume without the sensible qualities that quantity is subject to in nature. The reason, already mentioned, is that quantity is prior to sensible quality, as surface is to colour. Now, although we may consider quantity apart from sensible quality, we do not mean that it can also be that way, in a state of separation from sensible matter, outside the mind.

At this juncture a question is raised concerning the value of such abstraction as to truth. For if in nature there is no such thing of which we can verify the exact kind of triangle we define in geometry, nor the kind of homogeneity and unity required by number theory, how can we say anything true of triangle when truth is defined by the conformity of mind with what is ? If I say that Socrates is seated, and he *is* seated, what I say is true. But when I say that the plane triangle has its three angles equal to two right angles, how can this be true if a figure of that kind does not exist somewhere in the sense that Socrates exists ?

2. *Some Meanings of ' existence '*

Two things are to be considered in connection with this problem. First, that the terms ' to be,' ' being,' and ' existence,' each have several meanings, as we noted on an earlier page. If we take ' being ' for ' what is,' then both ' what ' and ' is ' each have several meanings. The single word ' what ' can be used to stand for the diverse things that Socrates is, according to what is intended by the questions we may ask about him, such as ' What is he ? ' ' What is his size ? ' ' What is his disposition ? ' ' What is his civil status ? ' ' What is he doing ? ' and so forth. Accordingly, ' is ' will not mean the same in the answers to these questions. ' What ' will again mean diverse things in the following questions and their answers : ' What is a billiard ball ? ' ' What is it made of ? ' ' What is it for ? ' ' What made it ? ' The adequate and proper reply to the first of the questions must include the answers to all of them. Now, what a billiard ball is made of, what its shape is, what it is made for, and what made it are plainly far from being the same *what*.

The term ' is ' or ' exists ' has likewise different meanings in the statements : ' Socrates exists,' ' Man exists,' . ' The equilateral triangle exists,' ' There is an equilateral triangle in the mind of Socrates,'

'There is a relation of reason called genus,' 'Some things are nothing at all,' 'Whatever is impossible is impossible', etc. Accordingly, 'being' can be said of all these things; whether they be thing or not thing, for 'thing' too has a large number of meanings.

To make plain what is intended by existence in connection with the abstract subjects of mathematics it will do to single out only a few of these meanings. 'Socrates *is*,' i.e. 'exists,' means that an individual called Socrates is alive. 'Man *is* an animal' implies that this is true whether there be an individual instance of the kind or not. 'Man exists' will be true if there is at least one individual, such as Socrates, who 'exists' in the first sense. But in 'Man is an animal,' neither 'man' nor 'animal' stand for individuals, nor for collections of individuals; 'is' stands for a composition made by our mind and true, regardless of whether 'Man exists' be true or not. 'There *is* a triangle with three equal sides' means that we can construct such a triangle; it does not imply that such a triangle exists as in 'Socrates exists' or 'Man exists.' 'One centaur *is* faster than another' refers to something that cannot exist outside the imagination — a pure fiction. That there *are* fictions would be false if intended to mean that they are in the way 'Socrates is,' 'Man is,' or 'The equilateral triangle exists.' 'There *is* a logical relation of genus,' merely means that our mind, comparing terms such as 'animal' to 'man,' or 'plane figure' to 'triangle,' forms a relation of one to many, such that the more universal term can be said of things which differ in kind. Though remotely based upon what is outside the mind, such relations cannot *be* outside it. As in the case of mathematical subjects, 'existence' is here related to our way of understanding.[1] Finally, even negation, as in 'non-being *is* non-being,' has being in still another sense of this term.[2]

---

1. "... Ipsa conceptio intellectus tripliciter se habet ad rem quae est extra animam. [a] Aliquando enim hoc quod intellectus concipit, est similitudo rei existentis extra animam, sicut hoc quod concipitur de hoc nomine "homo"; et talis conceptio intellectus habet fundamentum in re immediate, inquantum res ipsa, ex sua conformitate ad intellectum, facit quod intellectus sit verus, et quod nomen significans illum intellectum proprie de re dicatur. [b] Aliquando autem hoc quod significat nomen non est similitudo rei existentis extra animam, sed est aliquid quod consequitur ex modo intelligendi rem quae est extra animam; [i] et hujusmodi sunt intentiones quas intellectus noster adinvenit; sicut significatum hujus nominis "genus" non est similitudo alicujus rei extra animam existentis; sed ex hoc quod intellectus intelligit animal ut in pluribus speciebus, attribuit ei intentionem generis et hujusmodi intentionis licet proximum fundamentum non sit in re, sed in intellectu, tamen remotum fundamentum est res ipsa. Unde intellectus non est falsus, qui has intentiones adinvenit. [ii] Et simile est de omnibus aliis qui consequuntur ex modo intelligendi, sicut est abstractio mathematicorum et hujusmodi. [c] Aliquando vero id quod significatur per nomen, non habet fundamentum in re, neque proximum, neque remotum, sicut conceptio chimerae: quia neque est similitudo alicujus rei extra animam, neque consequitur ex modo intelligendi rem aliquam vere: et ideo ista conceptio est falsa." *In I Sent.*, d.2, q.1, a.3, c.

2. "Unde dicimus quod non ens *est* non ens. Quod non diceretur nisi negationi aliquo modo esse competeret." *In IV Metaph.*, lect.1.

Let us suppose for a moment that the word ' being ' had no more than one meaning, as the Ancient Greek Philosophers assumed — a position to which nearly all moderns have unwittingly returned. The following would then be unimpeachable : ' Man *is* a predicable species, and Socrates *is* a man ; he *is* therefore a predicable species, viz. predicable of any individual man ; it follows that Socrates *is* Plato, Ion, and each of these *is* Socrates.' Seeing there is always one sense or another in which anything or nothing *is*, if the diverse meanings of ' to be,' or of ' being,' were one and the same, the worlds of nature, of logic, mathematics, fiction, absolute or relative negations would in their turn be utterly one and the same. Whatever *is* ineffable, as well as what *is* namable, would *be* in the way that ' Socrates exists ;' and Socrates would *be* the way relations and negations exist.

Let us now revert to the existence of our abstract triangle. Like any other mathematical subject it is an *opus*, a work, in the broad sense of this term (*communiter loquendo*), yet at the same time a definable nature : ' what a triangle is ' follows from our own, human mode of understanding. This *consequi modum intelligendi* does not imply that mathematical subjects are embedded in our mind in the way of *a priori* forms ; we actually construct them, yet as a result we are faced with strictly definable natures. All the same, they differ from second intentions. These are formed by acts of comparing, which may likewise be called a ' making ' in the wide sense of this term. But whereas in mathematics the purpose of making is for the sake of knowing the subjects made, in logic the works are produced to set order in our mind, but for the sake of knowing subjects other than these, such as those of mathematics and natural science, pursued for no other purpose than to know. Logic is strictly an *organon*, a tool. Mathematics is more than that, though it takes on the nature of means when applied to nature.[1] The term ' existence ' as applied to relations of reason and mathematical subjects does not mean the same kind of existence, yet the same word is used because the existence of triangle, for instance, and that of genus are one in proportion.

We must now look more closely into the nature of mathematical abstraction as understood by Aristotle and St. Thomas.

---

1. An intellect which knows things independently of experience and whose means of knowing are prior to the things known, sees whatever truth which we possess in composing, dividing and demonstrative reasoning, but whithout composing or dividing or demonstrating. Such an intellect stands in no need of abstraction, and for this very reason it forms no second intentions, nor does it construct in order to know what we learn by construction in mathematics. Separated substances are neither logicians nor mathematicians.

XV. TO DEFINE WITHOUT SENSIBLE MATTER THINGS THAT CANNOT BE WITHOUT IT OUTSIDE THE MIND

We have seen, in a very general way, what is meant by things which cannot be defined without sensible matter. Let it be granted, for the moment, that they are what the science of nature is about. We already pointed out briefly that there is still another mode of defining, as when we define a number without any reference to a corresponding number of sensible things, or a figure without reference to the figure known to sense. This second type of definition differs radically from the first. The first was abstract in the sense that we left aside the individual sensible thing, like the bones and flesh of Socrates, but did retain bones and flesh ; for, without these, man can neither be conceived nor exist. By ' exist ' we mean that man could not exist even in the mere sense of truth ; since ' what it is to be a man ' is to be of bones and flesh, and no propositions about man as such are true which do not so consider him. The second is abstract in the sense that the definition disregards both individual sensible matter and common sensible matter. This, then, is a very different way of abstracting from matter.

1. *Abstraction of form, i.e., of quantity from sensible matter* [1]

In the first type of abstraction, the initial step is from the individual Socrates, Plato, etc., to man in general. After this first step, it is an easy progress from man to animal, from animal to living being. It should be noted, however, that it is the first step which is crucial, for by it alone we pass from the potentially intelligible to the actually intelligible. The further transitions — from man to animal to living thing — take place on the same plane of actual intelligibility : man, animal, and living being still requiring sensible matter in their definitions (at least until some proof is advanced that there can be living things without sensible matter). Now, to appreciate how entirely different is the second kind of abstraction, we have only to consider the example of circle, viz., ' a closed plane curve such that its circumference is at every point equidistant from the point within called its center.' That the status of this thing is very different from that of a chalk-circle on a blackboard, or the circular path of a planet, is plain from the fact that we cannot possibly verify the definition in experience. Even though we may have started by drawing a circle with a compass, the definition is not of what we have drawn. The drawing is no more than a stepping-stone to the goal of the true circle, and one to which we cannot return, once the circle is constructed and defined, although we may appear able to do so. And the same holds for ' sphere ' which abstracts, e.g., from ' bronze sphere.' That is why St. Thomas insists

1. Cf. *In Boethium de Trinitate*, q. 5, a.2.

that mathematical subjects are not similitudes of things outside the mind.

Now, of the abstract sphere, it is to be noted, not only that it is neither hard nor soft, cold nor warm, nor coloured, but that it is not even a common sensible like the shape of the bronze sphere. When considering ' sphere ' in separation from everything *per se* or *per accidens* sensible, the mind confines itself to something that has the nature of form, not with regard to a matter incidentally sensible like the bronze of a bronze sphere, but with regard to a matter which is simply the tree-dimensional continuum of the sphere. In the abstract sphere, the continuum is as the matter, and the shape is the form. In other words, in order to arrive at the true geometrical sphere, the mind must completely abandon that reality which requires sensible matter in its definition. That it has indeed done so is manifest from the fact that neither the definition of the mathematical sphere, nor any proofs or reasonings derived from that definition ever need to be confirmed by comparison with natural objects. If the statement ' a sphere *is* a three-dimensional continuum bounded by one surface which is at every point equidistant from a point within called its center ' depended upon verification in experience for its truth, we could not know it to be true until we had made the verification. But the fact is that, in the very act of predicating the definition of the definitum, we *see* that the proposition is true : that there is such a body, that sphere *is* in the sense that we may form true propositions about it, whereas ' diagnonal commensurate with its side ' is not. This condition of things never applies to definitions or propositions about things that can be outside the mind, like man, or snubnose, for it is essential to the latter that there be possible instances of them in nature. Thus in the mathematical object we have an actual intelligibility of another kind, free from the limitations of sensible matter.

It is sometimes thought that mathematics is about common sensibles ; but this is wholly wrong, for these, too, like the proper sensibles, are *per se* sensible. Mathematics is about forms as the mind has abstracted them by construction, and not about what may vaguely correspond to them in the order of common sensibles. Both the sensible *per se* and the sensible *per accidens* can serve as their source ; it is abstraction as practised on individual or common sensible matter which leads by construction to the subjects of mathematics. Should we make our start from common sensibles, for example, from a triangle drawn on the blackboard, our science will still deal with what has been abstracted and not with what it has been abstracted from. The demonstration is not even about the *kind* of triangle that is drawn on the blackboard, for this is a white one, actually the shape of a mass of chalk hanging there, a *per se* sense object. There is indeed something about the chalk-lines that our mind can consider without them ; but this ' without *sensible* lines and angles ' is not understood to *be* in the

sensible lines and angles. If it had to be in them, it could only be so in the way in which a sensible triangle is a triangle, viz. in sensible matter and without any verifiable exactness.

There is, then, a separability to mind that is typical of quantity, of number or dimension. The reason is that, in the things of nature, quantity is prior to quality, as surface is prior to colour ; and prior to the *per accidens* sensible subject of sensible quality as such.[1] So that, even if quantity cannot exist in reality without a sensible subject and without sensible quality, it can nevertheless be abstracted from them and considered in the way outlined above.

2. *Whether there can be an abstraction of quality in the way that there is one of quantity*

But there can be no abstraction of sensible quality in the way that there is of quantity. The best way to explain this will be by facing the objection that, when we abstract from sensible quality we ought still to have quality, just as when we abstract from sensible quantity we still have quantity. When we say ' Socrates is wise,' for example, wise is predicated as a quality that is not *per se* sensible. Why not a special science of quality, then, like that of quantity? Two things should be noted in this connection. First, that what we are examining here are the various modes of definition inasmuch as they distinguish the sciences in kind. Second, that the properties of things that are not some way or other in sensible matter are not positively known to us until they have been demonstrated ; and if such a demonstration is possible, it will show that there is still another mode of definition. Further consideration of these two points will allow us to appreciate how unique is the case of quantity.

3. *The formal distinction between sciences is not based per se upon degrees of generality*

Regarding the first mode of defining, we must observe that the sciences are not distinguished according to each and every kind of abstraction. We noted that from the universal ' man ' we can go on to ' animal,' which is more universal ; and from here to ' living being,' and hence to ' being,' and then to ' whatever can have the nature of object ' including even ' that which cannot be an object in any sense.' These are degrees of sheer generality, — specific, generic, or proportional — and the degrees lying within each are inexhaustible. If this sort of abstraction could distinguish the sciences, there would be as many sciences as there are degrees of generality. Besides, they who define metaphysics by nothing more than generality would find this generality superseded by a far greater one ; for what could prevent us

1. *In II Physic.*, lect.3 ; *In Boethium de Trinitate*, q.5, a.3.

from saying ' let A stand for the subject of every such a science including that of metaphysics,' or ' let B stand for what is impossible as well as for its opposite.' The art that does not name anything at all, viz. logistics (*logistikè* : the art of calculation), would be queen. But to the true nature of science and its true mode of definition all this kind of thinking is totally irrelevant. A degree of generality is not more actually intelligible in the measure that it is more general, but in proportion as it is removed from matter. No definitions contain individual sensible matter ; some contain common sensible matter ; others, like those of mathematics, abstract from common sensible matter too, even though the defined could have no being in nature without it. And if there were definition without sensible matter of something that could also be in reality without it, we should have a third mode of defining, and therefore a third kind of principle of science, or degree of actual intelligibility. So soon as we establish that there are objects like spheres or equilateral triangles, we have shown that there is a peculiar mode of definition which deals with them. In the same way, if we could demonstrate that there exists a reality without sensible matter at all, we would know that there was a third mode of definition and that it held good for what is in the way of Socrates.

That a degreee of generality does not carry with it more actual intelligibility can be seen from the fact that, in knowing man only as an animal, we do not know him distinctly as a man, for the elephant too is an animal. The general, here, is more potential and confused, whereas the perfection of knowledge lies not in the direction of the more general but rather in the direction of something less general which must include nonetheless the more general, in the way man includes animal.

Anything is best known when known according to its own kind, the kind which can no longer be divided into other definable kinds. And although the definition of animal (' a body able to sense ') differs from the definition of man (' an animal able to reason '), as being more general, they do not differ as to mode of definition, for both are with sensible matter. This may perhaps become clearer if we notice that, even when we consider Socrates as *this man*, or as *this animal*, as *this living being*, or as *this thing*, our degreee of generality is widening, but we are always pointing to the same individual matter as attained in sensation. Similarly, whether man be defined as man, or animal, or living being, common sensible matter enters into each definition.

XVI. THE TRUTH OF A THIRD MODE OF DEFINING IS NOT SELF-EVIDENT

Now regarding the second observation, that there is a definition of ' life ' or of ' living being ' which does not include sensible matter, could only be shown by an *a posteriori* demonstration (i.e. from effect

to cause). We would have to prove that there can or must *be* such a thing, a reality without sensible matter ;[1] for there can be no way of learning that there is a reality of this kind except by a proof *that* it is, and that sensible matter does not pertain to what it is.[2] Hence, that there *is* a third mode of defining is a matter for demonstration. This is the peculiar condition of the third mode, if there is one.

1. *Defining quality without sensible matter*

If we did attempt to define quality without sensible matter, we could not succeed until we had also demonstrated that there must be quality of this kind. We actually do this when we demonstrate that there is a triangle whose three sides are equal, for the resulting figure is a quality. But it is a quality in abstract quantity, not a figure which is a common sensible ; in other words, it is considered as a figure without sensible matter at all. And why is it that in abstraction from sensible matter we can obtain qualities like straight, circular, etc. whereas we obtain nothing of the kind concerning ' quality not in quantity,' and could not do so without first proving that quality without sensible matter exists in reality ? In other words, why is it impossible to construct in the abstract a quality which would be related to proper sensibles in the way circle is related to the chalk-circle on the blackboard ?

The reason is that quality simply cannot be abstracted from sensible quantity and sensible matter, whereas quantity is easily abstracted from both sensible quantity, quality and matter (though not from all matter — as we shall see further on). The latter abstraction is possible because, on the one hand, what we call sensible matter is only sensible because perceived as the proper subject of sensible quality, and sensible quantity too is so called because attained through perception of sensible quality ; while, on the other hand, quantity is seen to be prior to quality : as surface is grasped as that in which colour is, or as three perceptible units, like three men, are known as that of which there are three (for each must be outside the other before this particular kind of order, known through perception of quality, can arise). Now, surface can be thought of apart from any sensible quality, such as hardness or colour, and defined as ' what is extended in two dimensions ' ; the number three, in its turn, is understood as the particular kind of order revealed by adding one to two, provided the elements are of the same nature. Prescinding in this way from sensible quality, we obviously prescind as well from quantity as sensible,

1. The mere fact that the possible expression ' a wholly immaterial substance ' reveals no contradiction does not entail that there can be such a substance.

2. Only then might we change the imposition of ' living,' or of ' living being', and make them analogical terms, as ' light ' is used analogically of both candle-light and the light of mathematics.

though not from quantity as such ; for we still have something extended in two dimensions, and still have three units that are one three. But if we detached, or thought we could detach, quality from *sensible* quality, we would at once lose our sensible quantity and sensible matter and what would we have left ? What would the abstract quality be ? We would be left with a mere expression, whose meaning could be susceptible of no more than the logical verification conveyed by the question : ' Is there an *immaterial* quality ? ' The point is that, though the question may have meaning, it does not answer itself. If the answer is to be that there does exist immaterial quality, such an answer calls for positive proof.

Unless we can demonstrate that there is quality apart from *sensible* quality, which would then be defined without sensible matter, we cannot know whether or not such a mode of defining is possible. When we speak of ' quality in the abstract,' we do not know exactly what we are talking about except logically. To hold for knowledge of anything more than the logical function of our term is to fall at once into a mental void. If, in the question ' what is man ? ' for example, the ' what ' were more than logical, the question would hold its own answer, and ' what man is ' would have to be taken as undefinable. In this state of affairs, on the other hand, ' squarable circle ' would be a definable nature for the simple reason that we can ask whether there is such a thing. To know how to ask a meaningful question would be the same as to know the answer, and the true meaning of the question would have to imply that that which the question is about must have more than a logical status and must *be*, at least in the sense of that of which there is more than a nominal definition.

2. *Quantity compared to quality in point of definition*

But none of this holds true for quantity considered apart from sensible quantity. First, quantity in the abstract, that is, quantity separated by the mind from all sensible qualities, can be so considered in separation no matter if it cannot actually be in this fashion ; nor does the act of abstracting it assert anything about its mode of existence in reality, because whatever we do affirm or deny of it never bears upon it except qua abstracted from sensible matter *in mind*, and never requires the supposition that it could have the kind of being that man has. And when we say ' in mind,' we do not mean that abstract quantity is of the mind exactly in the way in which the second intentions of logic, like the relations of universality, are formed by the mind.[1] The mind does not form the nature of equilateral triangle, although it forms a mental construction in order to reveal it. In

1. Although even here the mind does not form ' what it is to be a second intention ', any more than Socrates becomes a *per se* cause of ' what it is to be a man ' by generating one.

mathematical abstraction, the mind's only function is to make the separation, so that there is absolutely nothing to oblige the mind to maintain that what is thus separated in mind can, or should be, also separable in reality. When we demonstrate that there is a triangle whose sides are equal, we never imply that there is such a figure in reality, either with or without sensible matter. We merely show that there *is* such a definable subject in the sense of truth, and that whatever is demonstrated of it is true of it qua abstracted from all sensible matter.

But quality, we have seen, cannot be abstracted in this way. That the name ' quality ' could never be extended to something that is not sensible, or that is not the quality of quantity, is certainly not self-evident. But the lack of evidence for this identification is not a reason from which we may infer that such an identification is possible, and that there is a quality without any matter whatsoever. Evidence to show that we might at least consider such a quality would depend upon a demonstration that there is a third mode of defining, which means proving that there *is* such a quality in reality in the way that man is in reality. It would be evidence leading not merely to what might be considered in separation, but to what *is* separate in reality.

The free development of mathematics in its independence from sense experience is guaranteed by the very nature of quantity inasmuch as it is basically no more than repetition of the same ; whereas quality is not. And while, even of quality, there may be more of the same, this will depend upon quantity : to have more redness of a given shade we need a greater surface. Now, repetition of the same gives rise to various new kinds of form without bringing in anything from outside that which is the same, like $1+1=2$, $2+1=3$, etc., each unity obtained by adding a unit, being different in kind ; by multiplying any number of the series by two, we reach the series of even numbers ; and we can then show the properties of odd and even, and the properties pertaining to certain of their kinds. So the forms of numbers (like two-ness) and of dimensions (like straightness) arise from the different types of order that can be reached — as we prove by constructions — so long as more of the same can be had. And this leads us to another basic notion which we must now make clear : that of intelligible matter.

XVII. THE NOTIONS OF INTELLIGIBLE MATTER

If we are to understand the nature of the abstraction that is proper to mathematics, viz., of arithmetic and geometry taken in the traditional sense, we must examine what Aristotle calls ' intelligible matter.'[1]

1. *Metaph.*, VII, c.10 (St. Thomas, lect.9-11) ; *De Anima* III, c.4 (St. Thomas, lect.8) ; *Ia Pars*, q.85, a.1, ad 2 ; *Q.D. de Veritate* q.2, a.6, ad 1. An elaborate study on intelligible matter will appear in the next issue of this periodical.

1. *Mathematical individuals and their matter*

Aristotle begins his explanation of intelligible matter by calling attention to the fact that, even in the world of mathematics, there can be individual objects, like the individual circles we describe to construct a triangle whose sides are equal. Now these circles do not differ by what they are ; for one is as much a circle as the other, and they are even of the same in radius. The only difference between them is numerical. The same holds for numbers : we may have as many instances of the same number as we please. Now, ' what circle is ' is not the same as to be ' this particular circle,' e.g. the one to the left of that other. If it were, there could be only one circle, and circle would be this single circle. The individual circles are not part of the definition of ' what circle is,' while the definition is verified in each and every one of them ; nor are the instances of ' two ' part of ' what two is.' [1]

There is therefore something about a particular or given circle which has nothing to do with ' what circle is.' In fact, as particular or given, a circle cannot be defined any more than that real individual, as individual, which we discussed in an earlier chapter. *This* circle, as such, can only be designated ; no name can be given to it, although it may be convenient to use a symbol in its place — as A might serve to distinguish this one from another, B. In other words, an individuating principle is here at work, a principle analogous to that already pointed out in our bowling pins. Of the pins, we concluded that we could have many of the same, because we had enough material. The possibility of ' many of the same ' was to be attributed to *this* matter that the bowling pins were made of.

Now the ineffable individuals of mathematics, like those of physics, must require something extrinsic to ' what ' they are to distinguish them from one another, some subject analogous to the designatable matter of the bowling pins. Yet there is a profound difference. In the first case it is *this* individual sensible matter. The latter too is a *this*, and in the nature of matter, but not sensible, for we neither can, nor need verify it in sense experience. The mind nevertheless does reach it, inasmuch as we are quite clear about ' two or more circles of the same radius,' even though we could never designate them to external sense. So we call the matter of these mathematical individuals ' intelligible', in the sense that it can be reached only by mind, and is not the individual matter of external sense experience.

---

1. Unless we interpreted the term ' circle ' to mean the collection of all circles, as ' two ' can be taken as the class of all couples. This would actually be an interpretation of the symbol 2, not a definition of ' what two is,' and therefore not even an interpretation of the *name* ' two.' What we have in mind rather is the number two, that " metaphysical entity " of Lord Russell's, " about which we can never feel sure that it exists or that we have tracked it down." *Introduction to Mathematical Philosophy*, p.18.

There is a further analogy between sensible and mathematical individuals. The first, like Socrates, are known in actual sensation in such a fashion that when the sensation ceases, they too, so far as the sense is concerned, cease to be. But, even when the man Socrates himself ceases to exist and sensation of him ceases to be possible, the mind's conception of man, ' what man is,' whether the knowledge be confused or distinct, remains unaffected. Indeed, of the conception it may be further asserted that even if no one were considering it, or if all minds capable of considering it were to perish, the notion, ' what man is,' would remain unaltered. While Socrates *is* in a sense in which ' what man is ' is not, he could not be in the sense in which he *is* if it were not true that ' man *is* a rational animal. ' It may be that something comparable takes place when I cease to consider *this* individual circle that I bore in mind while drawing a circular figure on the blackboard. Whatever it may be when I no longer consider it, it never is in the way in which the circle is ; for the circle is no less what it is when I cease to consider it.

2. *Why this individuating matter should be called intelligible* [1]

But this does not make wholly clear why such individuating matter should be called intelligible, rather than ' mental,' or ' mind-stuff'; terms which would leave it conveniently vague, yet distinguish it sufficiently from sensible matter. Further, of the mathematical individuals Aristotle says that, when we do not actually consider them, " it is not clear whether they exist or no " ; while St. Thomas, in his commentary, refers them to what Aristotle elsewhere calls the passive or ' corruptible intellect.' [2] Again, since individual real things are known only when actually sensed or actually in the imagination,[3] it seems that we should call this individuating matter by a similar term, like ' imaginable,' for the imagination is what ' corruptible intellect ' seems to mean. But while this last remark is certainly true, we believe that there is nervertheless good reason for the expression ' intelligible matter,' just as there is good reason for speaking of a corruptible intellect.

By intellect we mean the power which grasps what a thing is, like ' what circle is,' and the power which asserts what a thing is or what it is not — whether the knowledge be confused or distinct. Such acts belong to the intellect and to no other power. But, when intellect asserts that this man Socrates exists, to be in conformity with what *is* in the way Socrates is, intellect must depend upon an actual, external,

1. Cf. St. Thomas, *In Boethium de Trin.*, q.5, a.3, c., ad 3 et 4.

2. *Metaph.*, VII, c.10, 1036 a 5. *De Anima*, III, c.5, 430 a 25.

3. St. Thomas, *Metaph.*, *ibid.*, lect. 10, n.1495.

sense perception, for the reason that this kind of individual can only be attained with dependence upon external sense. Yet, though Socrates may now have left the place where we are and where we saw him, and though he may now even have ceased to exist, we still hold the individual image of him in imagination ; this individual image can still be attained when Socrates is no more to be seen or heard. And there is something else about this image that is peculiar to it : it can be multiplied at will, to look somewhat like the series of images reflected when Socrates stands between two mirrors. We can imagine a crowd of individual Socrates even if there is no such crowd in fact. The crowd is made up of imagined individuals, imaged as individuals. In short, there is a freedom here that is not to be found in external sensation. Instead of Socrates, we might have chosen the instance of a circle drawn on the blackboard, to multiply in imagination as we will. Observe, however, that this image does not represent a mathematical individual, for it is the image of a visible white figure on the blackboard. Even though we may imagine as many white circles on the blackboard, or as many hard bronze spheres as we wish, we must remind ourselves that mathematical individuals, the individual circles or spheres which we use in mathematics, are neither white nor black, warm nor cold, hard nor soft ; because nothing of what they are, nor of what we assert of them implies sensible matter. If it did, they could not be exact as we know them to be. In this respect, we know that they are not like the circles gathered in imagination from the blackboard. They are like them only in the respect that they, too, are many. But are they likewise seated in the imagination ?

3. *Mathematics and the imagination*

When intellect asserts something about this circle A, e.g., that *it* is a circle — as circle is mathematically defined — and that *its* radius is equal to that of the circle B, intellect then refers to an individual owning something of the status of the imaged individual Socrates, except that this individual is not taken from external sense experience. (Note that even the image of Socrates that I now have in mind is a *this*, though not a *this* that I can point out to sense, while I could use this drawing of a circle on the blackboard to bring my thought to rest on the one I have in imagination.) Now, there are two things to be noted. First, when dealing with its proper object, intellect does not concern itself with the individual ; but, secondly, intellect can make statements about the individual which are true. For example, in grasping what circle is, and in asserting whatever is true of circle, intellect does not attain an individual ; but, of a given circle A, intellect may truly assert that it is a circle, and that its radius is equal to that of B. If follows, therefore, that in making statements like the last two, intellect must be depending upon a power of mind which

perceives individuals directly, as is done in the imagination — upon some internal sense.

Why should the individuating matter of mathematical objects sometimes be called 'imaginable,' sometimes 'intelligible'? Individual circles are in the imagination, and in it have their being as *this* circle and *that* circle, here and now, in *this* organic power, that is no less corruptible than the external senses like sight or hearing. However, they would not be at all if it were not for intellect which summons them; and this intellect could not do without the imagination — unless it acquired a completely new mode of knowing, one not natural to intellect as we know it. It is plain, then, in what sense we speak of 'corruptible intellect.' In no way does it imply that imagination is intellect, not that the intellect itself is corruptible. It only means that, in the representation of mathematical individuals, imagination and intellect are interdependent. The two circles are summoned by and for the purpose of the intellect, viz. demonstration, and only the intellect can *verify* that they are circles. The imagination itself does not do this. On the other hand, if the imagination were destroyed the intellect would lose the exercise of its power to attain the individual; even as the imagination would be powerless to represent mathematical individuals if the defining intellect did not direct the representation.

4. *Mathematical universals and their matter*

The intelligible matter which we have so far considered is plainly not part of the definition of *what* the mathematical individuals are, e.g., of what a given circle is, for whatever is part of a definition is not an individual part. Nevertheless, what mathematical individuals are, and what they have in common cannot be defined without including something having the nature of matter, something proportional to the bronze or wood of a sensible sphere. For when we have abstracted sphere from sensible matter of whatever kind, the mind still retains something that is in the nature of matter, the matter of the abstract sphere, viz. the three-dimensional continuum of which sphericity is the form. No mathematical entity can be considered apart from a subject, like triangle apart from its lines, or the number three apart from the three units. The continuity of the line is the matter of the circle, and the figure of the line is its form. The three units are the matter of the number three, whereas the oneness that is peculiar to three as distinguishing it from any other whole number, is its form.[1]

1. For the number three is not the same as three units or three ones. Number is a plurality measured by the unit, the indivisible 'one' which is the principle of number. Now the measure must be of the same nature as the measured: measure and what is measurable by it must be homogeneous; the standard of length is a length; of weight, a weight. There is, however, something peculiar to number, inasmuch as the measure of a plurality is not necessarily itself a number, viz. the indivisible 'one' which is a perfect

If, per impossibile, the figure we call ' equilateral triangle ' were ' what it is ' apart from the lines, that is, if it were definable without them, then ' to be an equilateral triangle ' and ' to be this particular individual one ' would be entirely the same. It would be ' what it is ' in abstraction from all intelligible matter. The same would hold for man, if he were definable without sensible matter : what man is would be wholly the same as what the definition expresses only as form ; so that if this thing so defined were to exist as Socrates does, ' to be man ' would be incommunicable in such a fashion that there could be only one single individual man. The individual would exhaust the species, and any other individual would be different in kind.

In other words, whenever a thing owes what it is to something extrinsic to its form, then, to be what it is, is to be of matter and form, inseparably, such as triangle and straight lines, or three like units and one three. If three could be the one three that it is, without the three units which are its matter, the oneness peculiar to three would be destroyed.

5. *Mathematical science defines with intelligible matter*

Hence, both in mathematics and in nature, the ' thing ' and ' that by reason of which ' it differs from another in kind are not wholly the same. And this is because of the matter essential to their definition, i.e. to ' what they are.' If there were a thing which is ' what it is ' irrespective of all matter, then the thing and what it is would be quite identical ; it would be individuated by its form. Let us repeat, however, that whether there is such a thing existing as Socrates does, but in separation from matter, would have to be proved. Only then

---

measure in the sense that it is used to express exactly and completely that of which it is the measure. What is essential here is that the measure-unit should be wholly of the same nature as the elements of the measured. Otherwise a number would be no more than an aggregate, a collection ; it would be all that it is in its matter alone. Now, from the viewpoint of calculation it is indeed no more ; so that, if we define numbers by no more than the operations which can be performed on them, 1 and 0 are just as much numbers as two and three ; and fractions, irrationals, and the rest, will be special instances of number, inasmuch as they are interpreted in terms of the properties of certain operations that can be performed on them, and which they share with the more familiar instances of mathematical entities. (Cf. COHEN and NAGEL, *An Introduction to Logic and Scientific Method*, chap.VII). For strict calculation, it is quite indifferent whether the units symbolized by 3 are of the same nature or not, that is, whether we refer our 3 to three bowling pins, or to the heterogeneous collection of ' a man, a centaur, and a logical intention of genus.' The art of calculation would indeed be very much restricted if it were to be no more than an instrument of mathematical demonstration. Fortunately it applies far beyond the limits of mathematical science, to things whose nature may be quite unknown, and applies as well to unknown operations, as in the theory of groups, where the operations are as unknown as the quantities they operate upon. (EDDINGTON, *New Pathways in Science*, chap.XII). Calculus, in the broad sense of this term, owes its effectiveness to the very indifference of abstraction by way of symbolic substitution.

could we know, in a positive way, that there is a mode of defining without even intelligible matter, because we would then know *that* there is that kind of thing.

There is, accordingly, a relation between the matter which is part of the definition, and the matter which is extrinsic to it ; for if there is to be an individual, either in nature or in imagination, the individuality will owe itself to something extrinsic to what is expressed by the definition, viz., *this* sensible matter or *this* intelligible matter, making possible many things the same in kind. 'What man is' cannot be individuated by itself, but only in Socrates ; and 'what circle is' only incidentally in the circle A. That which individuates here is matter as quantified and designated, either sensible or intelligible ; and of this irrational principle there can be more and more without end.[1]

CHARLES DE KONINCK.

(*To be continued.*)

1. We might go on from here to distinguish the two kinds of universal intelligible matter, viz. that of number, and that which is the continuum, showing how they lie at the basis of the distinction between arithmetic and geometry. For geometry is less abstract than arithmetic in its very mode of defining, the continuum being intrinsically indefinite by reason of its unlimited divisibility, thus being more in the nature of matter than number is. But this would carry us beyond our present scope. (Cf. ST. THOMAS, *In II Post. Anal.*, lect.9, n.5.) On what is meant by the arithmetisation of the continuum and how it is to be understood, the reader may consult Herman WEYL, *Philosophy of Mathematics and Natural Science*, chap. II.