Laval théologique et philosophique

Descartes : Mathematics and Sacredness of Infinity

Adam Drozdek

Volume 52, numéro 1, février 1996

Gregory Baum et la théologie critique

URI : https://id.erudit.org/iderudit/400977ar
DOI : https://doi.org/10.7202/400977ar

Aller au sommaire du numéro

Éditeur(s)

Faculté de philosophie, Université Laval

ISSN

0023-9054 (imprimé)
1703-8804 (numérique)

Découvrir la revue

Citer cet article

Drozdek, A. (1996). Descartes : Mathematics and Sacredness of Infinity. *Laval théologique et philosophique*, *52*(1), 167–178. https://doi.org/10.7202/400977ar

Laval théologique et philosophique, 52, 1 (février 1996) : 167-178

DESCARTES : MATHEMATICS AND SACREDNESS OF INFINITY

Adam DROZDEK

RÉSUMÉ : Selon Descartes, l'infinité est l'essence même de Dieu et, comme telle, elle est un attribut sacré que la science, en particulier la mathématique, ne peut appliquer à aucun objet. C'est le caractère sacré de l'infinité qui a amené Descartes à éviter d'avoir recours à l'infini en mathématique, bien qu'il ait manifesté son habileté à le faire en s'attaquant au problème que lui soumit Florimond de Beaune. Néanmoins, la connaissance de l'infini, parce qu'elle est vraie, est le fondement de toute autre connaissance ainsi que l'affirme le soi-disant principe de Descartes.

SUMMARY : According to Descartes, infinity is the essence of God, and as such, it is a sacred property not to be ascribed by any science, in particular, by mathematics, to any object. This sacred position of infinity was the main reason for which Descartes avoided using infinity in mathematics, although he showed his skillfulness in this direction by tackling, for instance, the de Beaune problem. However, the knowledge of the infinite is a foundation of any other knowledge as very, which is phrased in the so-called Descartes' principle.

The seventeenth century, the time of emerging calculus, can be called the century of the infinitesimal. Generally, mathematics seems to refer, even in the simplest instances, to infinity, e.g., infinity of numbers, or infinity of line length, but in the seventeenth century infinity became the hallmark of mathematics. It was an age of astounding agreement between empiricist England and the rationalist continent, especially France, with respect to the use of infinitesimals and infinite series in mathematics. This is epitomized in the fact that the two creators of calculus, Newton and Leibniz, lived on opposite shores of the English Channel and that so many creative mathematicians at that time — such as Cavalieri, Roberval, Fermat, and Pascal, to mention only a few — with no second thought, referred to infinity. However, Descartes' approach was in disaccord with the prevalent views of his time. Mathematician and philosopher, he gave more thought than his contemporaries to such questions as, Does mathematics really deal with infinity ? Are these theorems valid, which seem to require a reference to infinity to be proven ? Are such proofs accept-

able ? The contention of this paper is that because, according to Descartes, God is infinite and God's actual infinity is tantamount to his perfection (AT vii 47),[1] infinity acquires the status of a sacred attribute which is reserved to God alone. But even if infinity could be found in nature, it could not be comprehended by any scientific means. Most interestingly, using the sacredness of infinity is undermined by its use in mathematics ; therefore, mathematics should refrain from such references even at the cost of becoming more complicated or less useful that it would be with making such a reference.

I

Infinity can be ascribed with certainty only to God and to our will. Infinity is the essence of God, and in the infinity of our will, we can find that we are created in the image of God (AT ii 628). However, our reason, our powers of comprehension are finite and limited, and since the will's decisions are determined by reason, we have no, so to speak, immediate access to infinity. We have to content ourselves with recognizing infinity without being able to explain it (AT iii 292 ; ii 138). We are finite (our reason, that is), hence we should refrain from problems which would require comprehension of the infinite (*Princ.* 1.26).

An effect cannot be more perfect that its cause (AT vii 40-1) ; therefore, if I, a finite and imperfect being, have in me the idea of an infinite and perfect being, then there must be a being who is the cause of this idea. It is God, who is "eternal, infinite, omniscient, all-powerful and the creator of all things" (AT vii 40). God "necessarily exists" (AT vii 45 ; vi 34), since I exist and I am not the cause of myself, and since I possess an idea of an infinite substance. Because "there is more reality in an infinite substance than in finite substance," therefore, "there is in me somehow in the first place understanding (*perceptionem*) of the infinite before the finite, i.e., [understanding] of God before myself" (AT vii 45). It was once observed that on this principle rests Descartes' philosophy.[2] This principle — which, in other words, means that the idea of imperfection presupposes the idea of perfection — was not expressed more explicitly by anybody before Descartes and for this reason it can appropriately be called the Descartes' principle.

The idea of infinity cannot be obtained by simple extension of infinity, as suggested, among others, by Gassendi. For example, there is an intuition that somehow we are able to indefinitely increase the series of numbers without reaching its end. However, this power of not terminating this process of increase cannot be of human origin, the human mind could not give rise to such a mechanism. Therefore, the fact that "in counting I cannot reach the largest of all numbers" implies the divine origin of the power allowing me "to think about a larger number than any number thought by me before," and that this power can only come from "a being more perfect than

1. AT indicates a reference to ADAM and TANNERY's edition of *Œuvres de Descartes*, Paris : Cerf, 1897-1913, and *Princ.* to *Principles of philosophy*.

2. Roger LEFÈVRE, *La métaphysique de Descartes*, Paris : P.U.F., 1959, p. 75.

myself" (AT vii 139). In other words, the possibility of progressing into infinity presupposes in us a faculty which can only be explained by infinity (cf. AT vii 365). It is thus an illusion that the concept of infinity can emerge when using finite means, because the distance between the finite and the infinite never decreases by mere increase of the finite, hence one cannot pass to infinity by simply progressing in series of numbers. I cannot be the author of infinity because it surpasses me, because in the sequence of numbers, I can always go further without being able to reach its end.

However, Descartes not only expresses the principle of priority of the infinite over the finite, he says even more. Not only does finitude presuppose the existence of the infinite, but also the idea of God's infinity is "the clearest and most distinct," and the clarity of the idea of infinity is not in contradiction with the fact that "I cannot understand the infinite [...] since in the nature of the infinite lies that I — as a finite being — do not encompass it" (AT vii 46). God is incomprehensible (letters to Mersenne from 1630) — we cannot comprehend (*comprendre*) him, but we know (*connaissions*) him. How do we reconcile these statements ?

Descartes makes a distinction between two cognitive acts which do not have to coincide. Infinity is not comprehended (*compris*), he says, but understood (*entendu*) (AT ix 89), we do not imagine or conceive (*imaginamur, nec concipimus*) God's perfections, including his infinity, but we understand (*intelligimus*) them (AT v 154). "It is sufficient for me to understand (*intelligere*) that God is not comprehended (*comprehendatur*) by me, in order that I understand (*intelligam*) God in the truth and such as he is" (AT v 356). We don't positively comprehend God and his infinity, but we positively know he exists. Incomprehensibility is, in fact, a positive mark of God's infinity, since if it were comprehensible, it would not be a genuine infinity, i.e., not perfection. In a similar manner, we cannot enclose in our arms a mountain, but we are keenly aware of its immensity. We cannot use the power of the thunder, but we are clear about this power. Therefore, incomprehensibility guarantees the truth of the idea of God, it does not hinder it ; "incomprehensibility is a positive manner in which the infinity reveals itself to a finite mind as it is" and "the infinite is intelligible for the very reason that it is not comprehensible".[3]

Although there is no doubt about God's infinity, the question of the presence of infinity in the world is not so clearly posed. Descartes seems at least to pronounce his ignorance about its presence in nature. Theoretically, it may well be that infinity is in the world, but it is not for us to detect it. In particular, what about matter ? Is it infinite or finite ? Is it infinitely divisible or not ? To answer such questions, Descartes invented (AT v 167) the concept of the indefinite which was introduced primarily as the means of solving the problem of the limitations of our cognitive powers.

Indefinite is something for which we cannot imagine having any limits. Being indefinite implies neither real limitlessness, nor does it imply possessing some limits. We are suspending our judgment with that respect and declaring our ignorance by

3. Jean-Marie BEYSSADE, "On the idea of God : Incomprehensibility or incompatibilities ?," in Stephen VOSS (ed.), *Essays on the philosophy and science of René Descartes*, New York : Oxford University Press, 1993, p. 87, 91 ; cf. AT vii 368.

labeling something indefinite. For example, "having no reasons to prove it and even being unable to conceive that the world has bounds, I — says Descartes — call it indefinite. But I cannot deny that there may be some reasons known to God, the reasons incomprehensible to me [indicating that the world is finite] : hence I do not decisively say that it is infinite" (AT v 52). Infinite is inherently limitless in the eyes of our reason. We cannot think about matter otherwise as having no bounds, but is it for our reason to say that it really does not have any bounds ? The limits of our imagination and the objective limits should be thoroughly distinguished. Similarly, we recognize the divisibility of matter as true, but its "way of coming to pass is inconceivable" (*Princ.* 2.34), which does not mean that it is impossible. We, the limited beings, cannot perform such a division, but for the infinite being it is possible. Hence, extension is indefinitely, but not infinitely, divisible by thought.

These examples indicate that Descartes uses a tri-partite division of the world : God who is undoubtedly infinite and the infinite will ; next, particular objects in the material world, which are finite ; and finally, many aspects of the world that defy imposition of any limits and hence have to be considered indefinite. But in reality, that is, from God's perspective, the division is bi-partite : God himself, who is essentially infinite, and everything else, which is finite, because God understands the world, numbers, etc., and even things greater than the world and the number (cf. AT v 167).

The distinction made between infinite and indefinite is, in Descartes' words, simply the result of "necessary caution" rather than "affected modesty" (AT v 274). It seems, however, that this caution was the result of modesty. We see that Descartes did not have patience with those who treated the concept of infinity very lightly. For example, it was J.B. Morin's main fault that in his book "he always discusses the infinite as if he had completely mastered it and could comprehend all its properties." And Morin is not an isolated case : "This is almost a universal fault." The right attitude is to "submit oneself to it [i.e., infinity] and not to determine what it is or what it is not" (AT iii 293). For this reason, we should not be troubled by apparent paradoxes concerning infinite numbers. Does an infinite number exist ? We cannot decide, since our mind is finite and an infinite number "would cease to be infinite if we could comprehend it" (AT i 147). What about an infinite line ? If it exists, it could be measured in feet and in yards, so that one measurement would be three times larger than other. But can one infinity be larger than another ? Why not, Descartes responds, after all, "what rights have we to judge" this fact with our finite minds ? Questions like this should not be answered, since "only those who imagine that their mind is infinite seem to be obligated to examine such difficulties" (*Princ.* 1.26). From that perspective, it is interesting to see how Descartes approaches other mathematical questions and to look closer at some specific solutions in which Descartes seems to refer to infinity. As mentioned, the seventeenth century made the reference to infinity in mathematics very natural, even necessary. Was an impact of philosophical views perceptible in Descartes' mathematics ?

II

One general problem which is common both to physics and mathematics is the problem of divisibility. Divisibility is only hypothetical, and since its actuality is neither clear nor distinct to the human mind, it should be suspended and treated in terms of the indefinite. However, Descartes is not always very particular in distinguishing infinite from indefinite. For example, when discussing a Zeno's paradox, he talks about infinite divisibility of a distance, which amounts to creating the sum $1/10 + 1/10^2 + 1/10^3 = \ldots = 1/9$ (AT iv 445-6). What is important is that although such a sum, or rather the number of its terms, is "supposed to be infinite," and although "it is supposed" that this division "has been done an actually infinite number of times," we realize that it is not real division, that it is only a supposition of something which somehow has been done, and the nature of the division is unexplainable. Because in what way could a finite mind perform such a division ? Such a supposition is only made for the sake of argument, since, importantly, the result is already known, namely that the sum adds up to 1/9 and that the horse from this paradox overtakes the tortoise. The problem with the Zeno's paradox is that "as it is assumed, the ninth part of one yard is an infinite quantity, since it is divided in the imagination into infinite parts." However, because imagination is, in fact, powerless to grapple with infinity, the assumption of an actually infinite division is only imaginary.[4]

The problem of divisibility is a problem of a general nature. But the influence of Descartes' understanding of infinity can be detected in more specific mathematical problems. Three such problems will be presented.

In the second book of *Geometry*, Descartes presents the method of using tangents to analyze curves. Drawing the tangent (or rather the normal, i.e., straight line perpendicular to a curve) at an arbitrary point on a curve is "the most useful and most general problem" in geometry. The method did not rely on physically drawing a line, but on the analytical method proposed by Descartes, which consists in translating geometrical problems into arithmetical forms. This method of analytical geometry was quickly challenged by Fermat, who showed that certain results can be derived by a much simpler method than used by Descartes. One such problem that illustrates Fermat's approach was finding the tangents to a parabola.

For a parabola defined by the equation $y^2 = kx$ (see Fig. 1),[5] we draw a tangent that touches this parabola at point y. A line parallel to the ordinate y crossed the tangent at y', so that, by the equation of the parabola, $y'^2 > kx'$, and therefore,

$$\frac{x}{x'} > \frac{y^2}{y'^2}.$$

From the similarity of the triangles we have

4. Cf. also Yvon BELAVAL, *Leibniz critique de Descartes*, Paris : Gallimard, 1960, p. 228-230.
5. Fermat and Descartes, in fact, both use a cubic parabola $y^3 = kx$, which leads to slightly more complicated formulas without changing the essence of reasoning.

$$\frac{y^2}{y'^2} = \frac{(x+|W|)^2}{(x'+|W|)^2},$$

which in conjunction with the last inequality renders

$$\frac{x}{x'} > \frac{(x+|W|)^2}{(x'+|W|)^2}.$$

Because the values x and x' are chosen to be very close to each other, this inequality can be transformed into a so-called adequality, that is, pseudo-equality, or approximate equality, and then simplified. The resulting adequality is $|W|^2 = x^2 + xh$, where $h = x - x'$. After the term with h is removed, the equality turns into $|W| = x$, which is the equation to find the point W in order to draw the tangent at point the (x,y) of the parabola $y^2 = kx$ (AT ii 6-10). Fermat does not elaborate much on the nature of the magnitude of h. He simply says that the term containing it should be removed from the equation. This may be justified by the fact that h is extremely small, i.e., that it is an infinitesimal. It cannot be zero, otherwise x and x' would be the same point. In modern terminology, we would say that h approaches zero whereby the adequality could be substantiated as a genuine equality.

Descartes found the result correct, but Fermat's method did not appear acceptable to him, although he admitted that this method is simpler than his own (AT ii 514). What Descartes found particularly suspect was the use of the magnitude h which was null and yet not zero and therefore could be omitted in the equation, "which seems to be done gratuitously" (AT ii 169, 323). The use of such an infinitesimal h and its subsequent deposition rendered Fermat's method inexact, even false and absurd, which one can invent "without ingenuity and by accident" (AT i 490). Fermat, in Descartes' opinion, with such a proof showed that "he found his rule only gropingly or at least he did not conceive the principles clearly" (AT ii 129). How could Descartes accept such an unclear and indistinct magnitude ? The fruitfulness of Fermat's method could not be used as an argument for its acceptability, since any method should be based on a firm, clear and distinct foundation. Such a foundation could be found, to be sure, in the algebraic approach of *Geometry*, and Descartes showed that he could prove Fermat's result without resorting to unfounded magnitudes (AT ii 170-3).[6]

The second mathematical problem in which Descartes came close to the infinitesimal calculus was the problem of finding the area of the cycloid which is the path made by a point of the rolling circle. The crux of the proof consists in establishing the equality of a semicircle and a cycloidal segment (see Fig. 2). The proof is accomplished by pointing out that all the intervals MN, horizontal to the diameter AB of the semicircle, are equal to all the corresponding intervals GP of the other figure : "This

6. It is noteworthy that Descartes does not exactly solve Fermat's problem, but partially Fermat himself is to blame since it was not clear which magnitude he attempted to maximize with his approach ; for details see Gaston MILHAUD, *Descartes savant*, Paris : Alcan, 1921 [reprint New York : Garland, 1987], p. 154-155 ; BELAVAL, *op. cit.*, p. 305-307.

proves that the area FKO is equal to the semicircle ADB for those who know that generally, when two figures have the same base and the same height and all the straight lines parallel to their bases which are inscribed in one, are equal to those inscribed in the other in similar distances, then both of them have the same area" (AT ii 261). It is important to stress that in this proof, it is irrelevant that there are an *infinite* number of intervals in the semicircle and the figure similar to it, since it is enough to know that *any* horizontal line of the semicircle is equal to one line in the part of the cycloid. To be sure, the number of such lines is infinite, even uncountable, but this does not change the result ; however, Descartes' proof is not founded on this information.

Nevertheless, continues Descartes, because this proof relies upon "a theorem which perhaps is not accepted by all, I will do it in the following manner" (AT ii 261). And here Descartes proposes another solution to the cycloid problem. By repeatedly dividing the ordinates and using the method of exhaustion, he shows that the corresponding triangles in the two figures have the same area (Fig. 2) : area(ADB) = area(FKO), area(AMD) + area(BDN) = area(FGK) + area(OKP), etc., *à l'infini*, whereby the figures have the same area. However, in spite of using the expression *à l'infini*, this proof is similar to the previous approach in that it really does not rely on the fact that there are *infinitely* many such triangles ; rather, it is important that *any* corresponding triangles from both figures have equal areas, and the fact that there is an infinite number of such triangles is secondary. Descartes would say that this number is indefinite and man is unable to determine precisely its magnitude, hence any explicit reference to this number would undermine the validity of the proof.

There is another assumption here, although unspoken : the area of a surface is a sum of intervals. This aspect of Descartes' proof, however, is de-emphasized, since he deals in terms of relations rather than in terms of summation, i.e., relation between an interval of a known surface and an interval of a surface whose area is to be determined.[7] In this way, there is no need to discuss the problem of the nature of infinity, or the infinitesimals.

The closest Descartes seems to come to utilization of infinitesimals is in solving the problem of Florimond de Beaune. This was the inverse tangents problem, which consisted in finding a curve using its tangents. That is, whereas before the common point of a curve and its tangent was to be found, which amounted to finding roots of an equation, now the points of tangency are assumed to be known, and the curve is to be determined, which is basically finding a line determined by a differential equation. In particular, de Beaune wanted to find a curve AV perpendicular to line AB at point A, such that

$$\frac{YV}{YZ} = \frac{b}{IV},$$

7. Cf. Jean-Pierre CLERO, Evelyne LE REST, *La naissance du calcul infinitésimal au XVIIe siècle*, Paris : CNRS, 1981, p. 58-61.

where b = BC (AT iv 229 ; see also Fig. 3). Since in Descartes' construction line AB corresponds to y-axis and line AC to x-axis, this equation in today's notation can be rendered as

$$\frac{dx}{dy} = \frac{b}{x - y}$$

which amounts to the construction of the logarithmic curve[8].

Assuming that AB is divided into m parts, and PV is composed out of $n < m$ parts, Descartes is looking for the ordinate Aα of point V and finds it to be bounded by two sums, as in

$$\frac{1}{m} + \frac{1}{m-1} + \mathrm{K} + \frac{1}{n} < \mathrm{A}\alpha < \frac{1}{m-1} + \frac{1}{m-2} + \mathrm{K} + \frac{1}{n-1},$$

and concludes that "by dividing AB into more parts, we can approach more and more, *à l'infiny*, true length of lines Aα, Aβ and similar lines and thereby construct mechanically the proposed line" (AT ii 516). This is very close to acknowledging the existence of infinity. This, however, is not the case on two counts. First, he notices that "in order to describe this curve AV, we should move two lines," one from AH moving toward BC, and another from AB descending toward PH. The intersection of these two lines is always on the curve AV. Here Descartes gives his disclaimer : "But I believe that the two movements are so incommensurable that one cannot be ruled by another, and therefore this line belongs to the number of lines which I rejected from my *Geometry* as being merely mechanic lines [and not geometric]." The reason is that the first line moves uniformly with linear speed, the second increases its speed according to equation

$$v_2 = \frac{dx}{dt} = \frac{b}{1 - t}$$

for $0 \le t \le 1$, from which $x = -b \ln (1 - t)$, and hence we again see that the curve is logarithmic.[9] Therefore, although for each m, the line Aα can be constructed using methods of *Geometry*, the curve AV cannot be found, since it would require to pass to infinity with m which is impossible for Descartes. His constructive methods are applicable to certain stages of creating the curve AV, but these methods cannot be extended into infinity, and hence this curve is beyond their reach. We can approach more and more the true length of Aα, but it does not mean that the length of this line will be reached, which rules out finding AV. Therefore, as Belaval summarizes it,

---

8. Equation $\frac{dx}{dy} = \frac{b}{x - y}$ can be transformed into a linear differential equation $\frac{dy}{dx} + \frac{y}{b} = \frac{x}{b}$ and solved so that

$$y = ce^{-x/b} + x - b \quad \text{for some constant } c.$$

To eliminate x, we make line AH the x-axis (and now we have skewed coordinates) whereby

$$y = ce^{-x/a} \text{ and because } x_{\text{old}} = \frac{x_{\text{new}}}{\sqrt{2}},\ y = ce^{-x/(a\sqrt{2})} - b\ , \text{ or after transformation, } x = -b\sqrt{2} \ln \frac{y+b}{c},$$

which indicates that the line under scrutiny is a logarithmic line.

9. Cf. Christoph J. SCRIBA, "Zur Lösung des 2. Debeauneschen Problems durch Descartes", *Archive for History of Exact* Sciences, 1 (1961), p. 416.

"what is in accordance with his inventive spirit appears to be contrary to philosophical spirit : his philosophy was an obstacle to the consideration of infinitesimals".[10]

The second reason why this proof rules out passing to the limit is the usage of the expression *à l'infiny* (or *a l'infini*). Descartes quite frequently utilizes this expression (e.g., AT ii 92, 180, 207, 249, 427), but it does not appear to have in any of these usages a strong philosophical meaning of proceeding to infinity ; it is rather an equivalent of *etc.* or *indéfiniment.* This expression is used the way an exclamation such as "Oh my God" is used without implying any theological views, or "Lord, have mercy" in situations indicating that the last thing in the mind of the beholder is God's mercy.

III

Descartes was so very close to the spirit of his time, when calculus was originated ; however, he never accepted the methods of calculus as legitimate. They were frequently more powerful and simpler than his analytical approach, but pragmatic justification was not enough for Descartes to accept them. The main reason was that mathematics and, generally, science occupied only a secondary position in Descartes' system. He was an ingenious mathematician, but mathematics was only one branch of the tree whose trunk was metaphysics, since "certainty and truth of all knowledge depend only on knowing the true God" (AT vii 71), and mathematics was interesting only as an exercise field for his method (AT vi 29). In his scientific activities he never forgot the primacy of metaphysics and theology, and if there was a conflict between science and metaphysics, Descartes did not have any doubt about which side to choose.[11] Descartes' science is saturated with theology, and by occupying himself with science Descartes wanted to show the relevance of metaphysics and theology not only to philosophy, not only to every day life, but also to such abstract areas as mathematics and physics.[12] All intellectual struggles have only theological goal, since "all those to whom God has given the use of reason have an obligation to employ it principally in the endeavor to know Him and to know themselves" (AT i 144). How similar this statement is to Augustine's, who set knowing God and soul as the goal of his philosophical quest — and nothing else (*Solil.* 1.2.7). This knowledge is not gained by starting with a clean slate — that would be an impossible task. That is why Descartes castigates scholastic philosophy for using the principle, "Nothing is in intellect that was not before in senses" (AT vi 38). The assumption about the existence of God has to be made before starting this cognitive quest, since his existence and his attributes are "the foundation of truth," and, moreover, clearness and distinctness are based on the existence of God and on that everything is derived from him (AT vi 38).

10. BELAVAL, *op. cit.*, p. 310.

11. Therefore, Descartes did not withdraw his physical views out of fear of being judged as Galileo, as frequently suggested, since as Charles Adam showed, he had no reason for fear, A. KOYRÉ, *Essai sur l'idée de Dieu et les preuves de son existence chez Descartes*, Paris : Leroux, 1922 [reprint New York : Garland, 1987], p. 5.

12. As it was once stated, mathematics was a vehicle which transmitted the certainty of the foundational "God exists" to the material reality, Wolfgang RÖD, *Descartes : Die innere Genesis des cartesianischen Systems*, München : Ernst Reinhardt, 1964, p. 142.

Hence, methodical doubt of the First Meditation was not really the point of departure, since we could not comprehend ourselves and our doubt without "somehow in the first place" comprehending God.[13] Hence, his proof of the existence of God is not proof, but rather a test of the consistency of his view on the nature of cognition with this assumption ; this proof is in reality the means of knowing better what we know and how we know it. Therefore, the famous Cartesian circle is not broken by assuming that, to Descartes, reason is autonomous,[14] but by assuming its dependence on theological assumptions. Therefore, as aberrant can be considered the view that passages in which Descartes makes reason dependent on God are considered sometimes an expression of "aberrant view".[15] This dependence is present all the time. In this sense it is true that "the idea of God forms the center of Cartesian doctrine",[16] and there is a great deal of truth in the emphatic statement that "never a philosopher appeared to be so full of respect to the Divine as Descartes".[17]

Infinity was *the* characteristic of God, and hence it had the status of a sacred attribute. Only God was infinite. Therefore, the world could not be infinite. It could not be finite, either, under the danger of contradiction. Therefore, although reluctantly, Descartes introduced the concept of the indefinite. The world was indefinite from our perspective, from the vantage point of God it was finite. Because grasping infinity is beyond our reach, methods used in mathematics could not be infinitistic, either ; such methods would be at best inefficient, because ungraspable, sacrilegious at worst. A serious attempt to consider the universe infinite would amount to its identification with God, hence the universe is at most indefinite. By calling lines, sequences of numbers, approximations, etc., infinite, we would elevate mathematics to the level of the divine, which is unthinkable. Therefore, in Descartes' view, by attempting to encompass infinity, mathematics reaches into the domain reserved for theology and metaphysics. For this reason, mathematical truths cannot be treated on equal footing with theological truths ; in particular, reference made in mathematics to infinity (indefiniteness) is not the same as such reference made in theology, since we comprehend mathematical truths but not the essence of God, which is infinity, hence the reality of these truths is lesser than the reality of God. Consequently, because we are able to comprehend these truths, they are basically the truths of finite nature.[18]

---

13. 'I think therefore I am' can be considered a concise rendering of a statement, 'I think therefore I know I am', i.e., my knowledge does not imply existence, but it presupposes it ; cf. also remarks in Detlef MAHNKE, *Der Aufbau des philosophischen Wissens nach Descartes*, München : Anton Pustet, 1967, p. 144-150. "Ego from which Descartes ascends to God is always ego having an idea of God," Ferdinand ALQUIÉ, *La découverte méthaphysique de l'homme chez Descartes*, Paris : P.U.F., 1987 [1950], p. 222. "God, in Descartes' view, is the first object of knowledge. He knows Him even before he knows that he has thoughts", Robert A. IMLAY, "Intuition and the Cartesian circle," *Journal of the History of Philosophy*, 11 (1973), p. 23.

14. Peter A. SCHOULS, "Descartes and the autonomy of reason," *Journal of the History of Philosophy*, 10 (1972), p. 322.

15. George NAKHNIKIAN, "The Cartesian circle revisited," American Philosophical Quarterly, 4 (1967), p. 255.

16. KOYRÉ, *op. cit.*, p. 3.

17. BAILLET, quoted after KOYRÉ, *op. cit.*, p. 4.

18. Cf. also É. GILSON, *La liberté chez Descartes et la théologie*, Paris : Alcan, 1913 [reprint Paris : Vrin, 1987], p. 102.

The use of infinity in mathematical proofs has to be unclear by necessity ; since we are finite, we cannot grasp infinity, therefore such proofs are not proofs at all. The fact that they lead to some useful results may be treated as at most a happy accident, but not as an ultimate and reliable proof. Mathematics, more than any other area of science, should use clear means, and such means exclude infinity. Hence, with some reservations, it may be agreed that "at no moment Descartes lost from his view the goal present at all times : to eliminate from different sciences all concepts which cannot be known only by reason, i.e., which are not objects of the clear ideas".[19] Descartes wanted to eliminate the infinite from science because the powers of reason pronounce it being beyond its reach and theology pronounces it sacred. The clear idea of infinity should suffice us, but we misuse the clarity of this idea when we attempt to use it. Knowledge of the infinite is not infinite, and such infinity of knowledge would be needed to tackle infinity in mathematics.

Descartes made his best efforts to purge infinity from mathematics by pointing to the gap between the object of mathematics and object of theology which is God. He attempted to purify the concept of infinity by reserving it to God and to his reflection in us, the will. Pragmatic reasons of efficient use of infinity were simply irrelevant to him as an argument for utilizing it in mathematics. In this sense we can agree with the statement that Cartesian philosophy is the "philosophy of infinite perfection of infinity".[20]

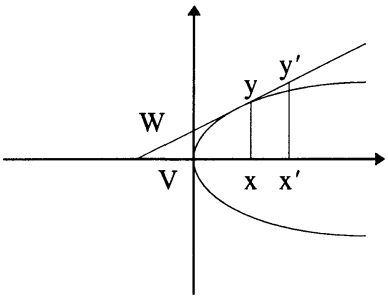

Figure 1

19. Pierre BOUTROUX, *L'imagination et les mathématiques selon Descartes*, Paris : Alcan, 1900, p. 33 ; "with some reservations," since Descartes' goal was theological, and knowing limitations of the mind was only a subsidiary goal.

20. KOYRÉ, *op. cit.*, p. 123.

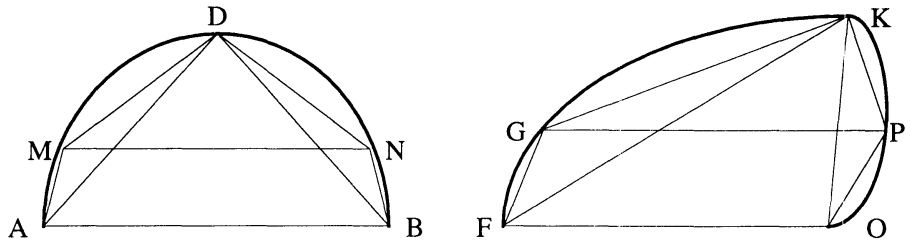

Figure 2

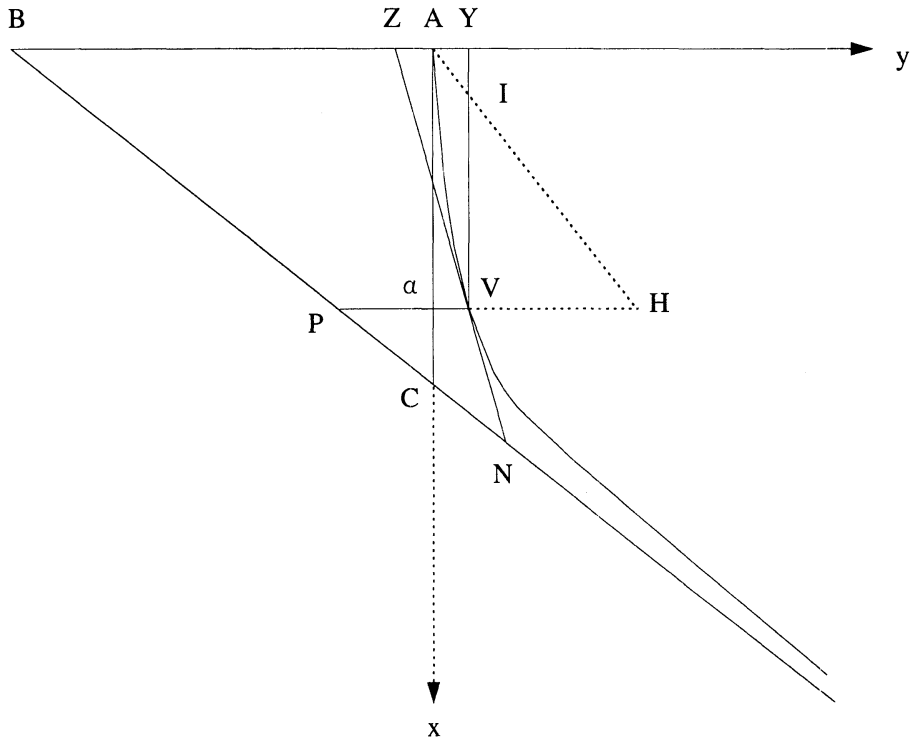

Figure 3